II. *Absolute Measurements of a Resistance by a Method based on that of* LORENZ.

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Communicated by R. T. GLAZEBROOK, *M.A., F.R.S.*

Received October 27,—Read December 11, 1913.

PRINCIPAL CONTENTS.

Published separately, March 19, 1914.

HISTORICAL INTRODUCTION.

THERE are many methods by which a resistance can be measured absolutely in the electromagnetic system of units, and all of these necessarily involve absolute measurements of a length and of a time. The length may be the axial length or radius of a coil, or the radius of a disc, or it may involve all of these, and the time may be the time of vibration of a magnet, or of a rotation of a coil, or of the period of an alternating current. In any case, the precision secured in the measurement of a resistance depends primarily on the accuracy obtained in these measurements of length and of time.

The first absolute measurements* of a resistance were made by KIRCHHOFF† in 1849, but it is to W. WEBER‡ that we owe the first distinct proposal (in 1851) of a definite system of electrical measurements according to which resistance can be measured in terms of an absolute velocity.

WEBER devised three methods by which the resistance of a wire can be determined absolutely, and he published the results of experiments by two of these. The first was by means of an earth inductor, and the second by observing the damping of a swinging magnet, the results obtained differing among themselves by 5 parts in 1900. In 1853§ he made a determination of the specific resistance of copper, but the experiments were made more to develop the methods than for exact measurements.

In 1862 WEBER|| made a more exact determination of resistance using a method compounded of his first two methods and eliminating the constant of the galvanometer. The results of these experiments were embodied in a determination of the value of the Siemens unit and of a standard coil sent to him by Sir WILLIAM THOMSON, but the unit obtained was about 8 per cent. less than the 1863 unit of the British Association Committee on Electrical Standards.¶

The measurements made by MAXWELL, FLEEMING JENKIN, and BALFOUR STEWART for the British Association Committee, although subsequently found to be incorrect by nearly 1·5 per cent., are the first with any claim to precision. The method adopted by these experimenters is that of the rotating coil and was devised by Prof. THOMSON (later Lord KELVIN) independently, we believe, of a prior suggestion by W. WEBER. The apparatus consists of a short-circuited coil rotating about a

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* Admirable summaries and criticisms of various methods will be found in WIEDEMANN'S 'Electricität,' vol. IV., p. 910; MASCART and JOUBERT'S 'Leçons sur l'Electricité,' II., p. 581; Lord RAYLEIGH, 'Phil. Mag.,' 1882; WIEDEMANN, 'Phil. Mag.,' 1882; GLAZEBROOK, 'Electrician,' 1890; DORN, 'Wissenschaftliche Abhand. der Phys. Techn. Reichsanstalt,' vol. II., p. 357, 1895.

† "Bestimmung der Constanten von welcher die Intensität inducirter elektrischer Ströme abhängt," 'POGG. Ann.,' Bd. 76, S. 412.

‡ 'Elektrodynamische Maassbestimmungen,' or 'POGG. Ann.,' Bd. 82, S. 337.

§ 'Abh. d. Kön. Ges. d. Wissenschaften zu Göttingen,' Bd. 5.

|| 'Zur Galvanometrie,' Göttingen, 1862.

¶ 'Reports of B.A. Electrical Standards Committee,' 1863.

vertical axis in the earth's magnetic field. Currents are induced in the coil and these produce a deflection of a small magnet suspended at the centre. The dimensions of the coil and the time of its rotation being known, the resistance of the wire of the coil can be calculated. This method was also used by Lord RAYLEIGH and Prof. SCHUSTER* in 1881, by Lord RAYLEIGH† in 1882, and by W. WEBER‡ also in 1882. The results are given in Table I.

In the method of the earth inductor due to W. WEBER a coil is mounted on a vertical axis with the mean diametral plane in the magnetic meridian. In circuit with the coil is a ballistic galvanometer through which a quantity of electricity flows when the coil is quickly turned through half a revolution. The resistance of the whole circuit can be calculated from the dimensions of the coils, the constants of the galvanometer, and the deflection produced by the half rotation of the coil. Methods based on this inductor principle have been used by F. KOHLRAUSCH and G. WIEDEMANN.§

In a third method used by KIRCHHOFF, two coils, between which there is a mutual inductance, are joined up in series with a battery and galvanometer, and a resistance, R, joins the junction of the two coils to a point on the circuit between the galvanometer and battery. The steady current deflection of the galvanometer is first observed and then the throw due to one of the coils being removed to a position in which the mutual inductance is zero. The dimensions of the coils and the constants of the galvanometer enable the resistance R to be calculated. Methods based on this inductive principle have been used by ROWLAND,|| GLAZEBROOK,¶ and MASCART, DE NERVILLE, and BENOIT.** Many of the developments of the method are of extreme importance, and, except for the essential principle being the same, there is little in common. Similar remarks apply also to the developments of other methods.

The method adopted by RÔITI†† and HIMSTEDT‡‡ is somewhat similar to that of KIRCHHOFF, but, instead of having to observe the deflection due to a single impulse, a constant deflection due to a series of impulses is obtained. The current through one of the fixed coils is made and broken n times per second, and the galvanometer circuit is made only on the make or break of the current. The method adopted by GUILLÉT§§ belongs in part to this class and in part to the method suggested by LIPPMANN.

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[*] RAYLEIGH and SCHUSTER, 'Roy. Soc. Proc.,' vol. 32, 1881.

[†] RAYLEIGH, 'Phil. Trans.,' vol. 173, 1882.

[‡] WEBER, 'Der Rotationsinduktor,' 1882.

[§] G. WIEDEMANN, 'Abh. der Berl. Ak.,' 1884.

[||] ROWLAND, 1876, 'Physical Papers,' pp. 145–239; The Johns Hopkins Press, 1902.

[¶] GLAZEBROOK, 'Phil. Trans.,' vol. 174, p. 223, 1883.

[**] MASCART, DE NERVILLE, and BENOIT, 'Ann. de Chemie et de phys.,' VI., p. 1, 1885.

[††] RÔITI, 'Nuovo Cimento,' III., 15, 1884.

[‡‡] HIMSTEDT, 'Berichte der Naturforschenden Ges. zu Freiburg i. B.,' Heft 1, 1886.

[§§] GUILLÉT, 'Journ. de Physique,' 8, pp. 471–477, 1899.

Another method used by WEBER is commonly called the method of damping. A magnet is suspended within a coil and set in oscillation (*a*) when the circuit is open, and (*b*) when the circuit is closed. The periods and logarithmic decrements are observed, and from a comparison of the results the resistance of the coil can be calculated. This method was used by WEBER and with important modifications by DORN,[*] WILD,[†] and KOHLRAUSCH.[‡]

In the method due to LORENZ[§] a metallic disc is rotated at a constant rate in a magnetic field produced by a current which circulates through a coil co-axial with the disc. The disc is touched at its circumference and centre by two wires, and the difference of potential is balanced against that at the extremities of a resistance, R, the current through which is the same as that circulating through the coil. When the mutual inductance of the coil and disc circumference, and the rate of rotation of the disc are known, the resistance R can be calculated. Methods based on this principle have been used by LORENZ,[||] Lord RAYLEIGH and Mrs. SIDGWICK,[¶] ROWLAND and KIMBALL,[**] DUNCAN, WILKES, and HUTCHINSON,[††] JONES,[‡‡] and AYRTON and JONES.[§§]

FOSTER,[||||] and afterwards LIPPMANN,[¶¶] suggested the use of a rotating coil, but contact with the extremities of the coil was made only at the moment when the induced voltage was a maximum. The induced voltage is balanced by that due to an external current (which may produce the field in which the coil rotates) through a known resistance. Observations by a method based on this principle have been carried out by LIPPMANN and WUILLEUMIER.[***]

The most recent determination of a resistance in absolute measure is due to A. CAMPBELL.[†††] In CAMPBELL'S experiments two very nearly equal alternating currents in quadrature, taken from a two-phase alternator of sine-wave voltage, are passed through a resistance R and the primary circuit of a variable mutual inductance respectively. The ratio of these two currents is measured by passing them through equal resistances and comparing the deflections on an electrostatic

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[*] DORN, 'WIED. Ann.,' 17, 1882, and 36, 1889.

[†] WILD, 'Mém. de l'Ac. des Sc. St. Petersburg,' tome 32, Nro. 2, 1884.

[‡] KOHLRAUSCH, 'Abh. der bayr. Ak. d. W.,' Bd. 16, 1888.

[§] LORENZ, 'POGG. Ann.,' 149, p. 251, 1873.

[||] LORENZ, 'WIED. Ann.,' 25, p. 1, 1885.

[¶] Lord RAYLEIGH and Mrs. SIDGWICK, 'Phil. Trans.,' vol. 174, p. 295, 1883.

[**] ROWLAND and KIMBALL, 'La Lumière Electrique,' vol. 26, pp. 188, 189, 477, 1887.

[††] DUNCAN, WILKES, and HUTCHINSON, 'Phil. Mag.,' p. 98, 1889.

[‡‡] JONES, 'Electrician,' p. 552, 1890. Also 'B.A. Electrical Standards Reports,' 1893, 1894.

[§§] AYRTON and JONES, 'B.A. Electrical Standards Reports,' 1897.

[||||] G. CAREY FOSTER, 'B.A. Electrical Standards Reports,' 1870.

[¶¶] LIPPMANN, 'Comptes Rendus,' 95, p. 1348, 1882.

[***] WUILLEUMIER, 'Journal de Physique,' 11, 9, p. 220, 1890.

[†††] CAMPBELL, 'Roy. Soc. Proc.,' 87, 1912.

voltmeter placed across either resistance. The voltage across the resistance R is balanced (by the help of a tuned vibration galvanometer) against the voltage induced in the secondary circuit of the mutual inductance. The value of the variable mutual inductance is found by comparison with a fixed mutual inductance whose value is calculated from its dimensions, and the resistance R is determined in terms of this inductance and the frequency of the alternating current.

Table I. gives the principal results, the values given in columns 5, 6, and 7 being those given by the experimenters. It will be seen that in some cases mercury standards of resistance were available, and in other cases the results are given in terms of the British Association unit (B.A. unit) or the Siemens unit. When mercury standards of resistance were available, the results (previous to 1892) state the length at 0° C. of a column of mercury having a uniform cross-section of 1 sq. mm. and a resistance of 1 ohm. The Siemens unit of resistance is the resistance at 0° C. of a column of mercury 100 cm. in length and 1 sq. mm. in cross-section; results which give the absolute value of the Siemens unit may therefore be reduced to give the length representing 1 ohm by taking the reciprocal of the absolute value and multiplying by 100.

In 1892, and again in 1908, the international ohm was defined as the resistance of a specified column of mercury. In 1892 Dr. VON HELMHOLTZ pointed out that a difficulty arose in determining the cross-section of a column of mercury owing to there being some doubt as to the correct value for its density. He suggested that the difficulty should be avoided by stating the mass of a mercury column of a given length which has a resistance of 1 ohm. This was agreed to and the international ohm was defined as the resistance at 0° C. of a column of mercury 14·4521 gr. in mass and having a length of 106·3 cm. The number 14·4521 is the product of 1·063 and 13·5956, the latter number representing at that time the mean of the best determinations of the density of mercury at 0° C. The cross-section of the specified column is therefore equal to 1 sq. mm. or nearly so. The ratio of the international ohm to the Siemens unit may therefore be taken as 1·063.

The B.A. unit is so much referred to in the earlier determinations that it may be useful to state clearly what is meant by the unit. In 1864 Messrs. MATTHIESSEN and HOCKIN constructed a number of coils of various materials to represent at certain specified temperatures resistances of 10^9 cm./sec. units of resistance as determined by the 1862–3 British Association Committee on Electrical Standards. The resistances of these coils did not keep absolutely constant, and in after years the B.A. unit was taken as the mean of the values of six of these coils at the temperature at which they were stated by HOCKIN to be correct. The B.A. unit of one period is not, therefore, necessarily the same as that of another period. Every precaution was, of course, taken to ensure constancy, but with wire standards of resistance great difficulty is experienced. In after years* it proved possible to trace the changes in these coils

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* 'B.A. Elec. Stands. Committee Report,' 1908.

with what appears to be a fair measure of success, and the corrections due to changes in the coils can in certain cases be calculated.

It follows, therefore, that if the B.A. unit of any particular period is known in terms of the resistance of a column of mercury, and if the coils used for the absolute measurements remained constant in resistance from the time of their measurements in terms of a mercury column to the time of their absolute measurement, the results given in Table I. can in all cases be reduced to give the length of the column of mercury having a resistance of 1 ohm.

But it is very probable that the resistance of many of the coils did not keep constant, and it is not possible for us to reduce the results except in a few cases. The instances referred to are the determinations by Lord RAYLEIGH, by Dr. GLAZEBROOK, by VIRIAMU JONES, and by AYRTON and JONES. In all of these cases comparisons were made with the B.A. standard coils, and the details of these comparisons have been preserved and published. In the Report of the B.A. Electrical Standards Committee for 1908, the changes in resistance of the coils used by Lord RAYLEIGH and Dr. GLAZEBROOK have been traced, and a comparison of the mercury standards of resistance made by Lord RAYLEIGH, Dr. GLAZEBROOK, and F. E. SMITH is given in Table VIII. of the same report. This comparison, together with the notes on the standards used, enables us to express Lord RAYLEIGH'S and Dr. GLAZEBROOK'S results in terms of the present mercury standards of the National Physical Laboratory. This we have done, the results being marked (S), while (A) indicates the results given by the author.

Referring first to Lord RAYLEIGH'S determination in 1882, we find that comparisons were made with certain B.A. unit coils and with mercury standards of resistance. However, the terminals of the latter were not at 0° C. but between 5° C. and 6° C., and it was shown by Dr. GLAZEBROOK in 1888 that an error of 24 parts in 100,000 was introduced because of this. If we apply a correction of this amount, Lord RAYLEIGH'S 1882 value of the ohm in centimetres of mercury becomes $106{\cdot}24\,(1+0{\cdot}00024) = 106{\cdot}26_5$, and the 1883 value becomes $106{\cdot}214\,(1+0{\cdot}00024) = 106{\cdot}239$. These values are given in Table I. (within 1 part in 10,000) as 106·26 and 106·24.

Dr. GLAZEBROOK'S determination of the ohm was made in 1882, and he constructed mercury standards of resistance in 1888. The principal resistance coil employed in 1882 was a platinum-silver coil known as "flat," and this also was used in 1888. In the interval it was assumed to have kept constant—there was at that time no certain evidence to the contrary. A careful survey of the history of the coils, which is published in the B.A. Report for 1908, shows, however, that "flat" increased in resistance in the interval 1882–1888 by 41 parts in 100,000. Dr. GLAZEBROOK'S value for the ohm in centimetres of mercury is 106·29, and this, when corrected for the change in the resistance coils, becomes $106{\cdot}29\,(1-0{\cdot}00041) = 106{\cdot}25$.

The coils used by VIRIAMU JONES in 1894 were compared with the B.A. standards by Dr. GLAZEBROOK. One of these coils was No. 3715, and its value in 1894 was stated by Dr. GLAZEBROOK to be $1\cdot00026_0$ ohms (international) at $14^\circ\cdot95$ C., the relation between the B.A. unit and the ohm (international) being taken as 1 ohm = 1·01358 B.A. unit.* From the results obtained at the N.P.L. in 1908 we conclude that this coil increased in resistance in the interval 1894–1908 by 7 parts in 100,000. Its value in 1908 was measured to be 1·00066 international ohms at $16^\circ\cdot0$ C. or 1·00034 international ohms at $14^\circ\cdot95$ C. When allowance is made for the rise in resistance of 7 parts in 100,000, it will be seen that the difference from Dr. GLAZEBROOK's value is 1 part in 100,000. We conclude, therefore, that the value given by VIRIAMU JONES in 1894 is not in error because of any uncertainty in the values of the resistance coils used.

Similarly we have investigated the coils used by AYRTON and JONES in 1897, and we find the values in Board of Trade ohms agree with the values in international ohms within 1 or 2 parts in 100,000. We conclude, therefore, that the length of the column of mercury representing the ohm is (from their experiments) $\frac{106\cdot300}{1\cdot00026} = 106\cdot274$ cm. Unfortunately, we are not sufficiently acquainted with the standards used by other investigators to reduce their results, and in the last column of the table the results given, except for the cases already dealt with, are those only in which mercury standards were available.

SECTION 1.—INTRODUCTORY.

The instrument described in this paper is the outcome of a desire expressed by the late Prof. J. VIRIAMU JONES at a meeting of the British Association Committee on Practical Electrical Units and Standards, in 1893. Prof. JONES expressed the hope that in the near future there might be constructed an apparatus based on the method used by LORENZ, which would be kept in constant use in a national laboratory and embody in concrete form a proper ultimate standard of electrical resistance.

In 1900 the Drapers Company of London promised to Prof. JONES the funds for the construction of such an instrument, and after Prof. JONES's death in 1901 the Company placed £700 at the disposal of the Executive Committee of the National Physical Laboratory in order that the instrument might be made.

The apparatus was to be in memory of Prof. JONES, and to be constructed under the superintendence of the late Prof. AYRTON and of Dr. GLAZEBROOK. Delay in proceeding with the work arose owing to the construction of the Ayrton-Jones current balance, and it was not until after completion of the balance in 1907 that a start was made. Unfortunately, Prof. AYRTON was in very poor health, and although keenly interested in the work he did not live to take any part in it.

The form of apparatus eventually decided on was considerably larger than anticipated in 1893. The metal work was much too heavy for machining in the

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* 'B.A. Elec. Stands. Committee Reports,' 1892 and 1894.

TABLE I.—Results of Absolute Measurements of a Resistance.

Date.	Observer.	Principle of method.	Resistance standards used.	Results given by author.			Value of ohm in cm. of mercury.
				B.A. unit in ohms.	SIEMEN'S unit (100·0 cm. of mercury) in ohms.	Ohm in cm. of mercury.	
1863	MAXWELL, JENKIN, and BALFOUR (B.A. Committee)	Rotating coil	—	1·000	—	—	—
1873	LORENZ	LORENZ	?	—	0·9337	—	—
1876	ROWLAND	Induced currents	Coils in B.A.U.	0·9878	—	—	—
1881	RAYLEIGH and SCHUSTER	Rotating coil	” ”	0·9893	—	—	—
1882	RAYLEIGH	” ”	” ”	0·98651	—	—	106·24 (A) *106·26* (S)
1882	H. WEBER	” ”	” ”	0·9877	—	—	—
1882	DORN	Damping of magnet	Coils (SIEMENS)	—	0·9482	—	—
1882	GLAZEBROOK	Induced currents	Coils in B.A.U.	0·98665	—	—	106·29 (A) *106·25* (S)
1883	RAYLEIGH and SIDGWICK	LORENZ	B.A.U. and mercury	0·98677	—	106·214	106·214 (A) *106·24* (S)
1884	ROWLAND	Induced currents	Coils in B.A.U.	0·98627 ± 40	—	—	—
1884	ROWLAND and KIMBALL	LORENZ	B.A.U. and mercury	0·98642	—	106·32	106·32 (A)
1884	MASCART, DE NERVILLE, and BENOIT	Induced currents	” ”	—	—	106·30	106·30 (A)
1884	RÔITI	” ”	B.A.U. and coils of SIEMENS and STRECKER	—	—	105·896	—
1884	WILD	Damping of magnet	Coils (SIEMENS and HALSKE)	—	0·94315	106·027	—
1885	LORENZ	LORENZ	Mercury	—	—	105·93	105·93 (A)
1885	WIEDEMANN	Earth inductor	”	—	—	106·265	106·265 (A)
1886	HIMSTEDT	Induced currents	Coils (SIEMENS and HALSKE)	—	—	> 106·01 < 106·16	—
1888	F. KOHLRAUSCH	Damping of magnet	Mercury	—	— Series 1 Series 2	106·405 106·274	106·3$_4$ (A)
1889	DORN	” ”	”	—	—	106·243	106·243 (A)
1889	DUNCAN, WILKES, and HUTCHINSON	LORENZ	Mercury and B.A.U.	0·9863	—	106·34	106·34 (A)
1890	WUILLEUMIER	FOSTER and LIPPMANN	Mercury	—	—	106·267	106·267 (A)
1890	V. JONES	LORENZ	”	—	—	106·307	106·307 (A)
1892	HIMSTEDT	Induced currents	”	—	—	106·259	106·259 (A)
1894	V. JONES	LORENZ	Coils from GLAZEBROOK	—	—	106·326	*106·32*$_6$ (S)
1897	AYRTON and JONES	”	Coils (Board of Trade)	(1 B.O.T. ohm = 1·00026 ohm)			*106·27*$_4$ (S)
1899	GUILLÉT	Induced currents	Coils (CARPENTIER)	—	—	106·214	—
1912	CAMPBELL	Alternating currents	N.P.L. mercury	—	—	106·27	106·27 (A)

The results given by the experimenters are indicated by (A). In a few cases it has been possible to express the results in terms of the mercury standards of resistance of the National Physical Laboratory. These results are indicated by (S). The method of reducing the results is explained on pp. 31–33.

Laboratory workshops, and Sir ANDREW NOBLE, F.R.S., was approached with a view to this part of the instrument being made in the workshops of Sir W. G. Armstrong, Whitworth & Co. at Elswick. Sir ANDREW NOBLE not only undertook that this should be done, but took such a keen interest in the work that he generously provided it at very much less than the cost price.

In preparing the plans we had full access to the working drawings of the McGill-Lorenz apparatus made for Prof. CALLENDAR by Messrs. Nalder Bros. & Co. Although we were unable to take much advantage of this opportunity owing to the big difference in the designs of the two instruments, we desire to express our thanks to Messrs. Nalder Bros. for their kindness.

The instrument herein described possesses many new features of importance. It is permanent, but determinations of resistance made from time to time will not assume constancy of any dimensions; it is comparatively easy to use, and the results obtained are believed to be of high precision.

SECTION 2.—THE METHOD OF LORENZ.

In this method, which was first employed by LORENZ* in 1873, a rotating disc is placed in a magnetic field produced by a current which circulates through a coil coaxial with the disc. In the apparatus used by LORENZ wire brushes made contact with the disc at its circumference and centre, and the circuit was closed by connecting the brushes, through a galvanometer, with the extremities of a resistance R. The voltage induced by the uniform rotation of the disc at n revolutions per second is Min, where M is the mutual inductance of the coil and a circle coincident with the edge of the disc, and i is the current through the coil. This induced voltage is balanced against the difference of potential Ri between the extremities of the resistance R through which the same current flows as through the coil. When equality of voltage is indicated by the galvanometer $R = nM$. M is calculated from the dimensions of the coil and disc and thus R is found. The coil used was of a large number of turns in order to make M as great as possible.

In the 'Philosophical Magazine' for November, 1882, Lord RAYLEIGH compares the method of LORENZ with other methods. After stating that he is disposed to consider LORENZ's method the best, he proceeds to deal with some of the difficulties. The first of these is the smallness of the resistance R which can be directly measured, and this led Lord RAYLEIGH to adopt a system of shunted resistances which, for part of our work, we also have used. Lord RAYLEIGH remarks that the influence of terrestrial magnetism and the thermo-electric effects at the sliding contacts are both very appreciable and give rise to trouble during the observations, but they can be eliminated by observing only the effect of reversing the battery current.

The more important portion of Lord RAYLEIGH's comments deals with the ratio of

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* LORENZ, 'POGG. Ann.,' 149, p. 251, 1873.

the dimensions of the coil and disc. The results are given of some calculations of the mutual inductance M of a coil of radius A and circle of radius a. The rates of variation of M with change in the diameter of the coil are calculated for various values of a/A, and it is shown that when a/A increases in value the rate of variation of M with change in a and also of A increases. Further, it is shown that, by using two coils separated to a considerable distance, it is possible so to proportion the radii and the distance apart of the coils that the error of mean radius of the coil does not affect the result; the diameter of the disc and the distance apart of the two coils are then the important quantities. Lord RAYLEIGH remarks: "It is clear that M vanishes both when A is very small and when it is very large; from which it follows that there must be some value of A for which the effect is a maximum and therefore independent of small variations of A." The same is, of course, true for the disc; by suitably proportioning the dimensions, the error of mean radius of the disc may be rendered negligibly small. This fact led Mr. A. CAMPBELL[*] to design a standard of mutual inductance in which the radius of the secondary is not required to be known with great precision; it also guided us in fixing the dimensions of our coils and discs so that the diameters of the latter need not be accurately known.

In the first and second series of experiments carried out by Lord RAYLEIGH the inductance coils were situated nearly in the plane of the revolving disc as in LORENZ'S original use of the method. In the third series the coils were separated from the disc to such a distance as to render the accuracy of the results practically independent of the mean radius of the coils. It is right to say here that in the design of the apparatus described in this paper we were largely influenced by Lord RAYLEIGH'S investigations and by Mr. CAMPBELL'S work.

Of the experimental difficulties noted by Lord RAYLEIGH the more important are: (1) troublesome thermo-electric effects at the sliding contacts notwithstanding that the edge of the disc was amalgamated with mercury, and (2) effects due to terrestrial magnetism. As before stated, both of these effects are eliminated by taking a sufficient number of readings with reversals of the current, but it is evident that good readings cannot be taken if the magnitude of the effects is subject to sudden fluctuation.

In 1890 Prof. J. V. JONES[†] made a number of suggestions towards a determination of the ohm. He had made a number of experiments in his laboratory at the University College, Cardiff, and stated that he was of opinion that, if apparatus were constructed on a large scale and with a certain perfecting of detail, a single set of observations would give a result accurate within 1 part in 10,000. In the electrical observations the principal difficulties which he had contended with were: (1) variations in the thermo-electric effects at the brush contacts, and (2) variations in the rate of rotation of the disc. A considerable reduction of the first difficulty was

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[*] CAMPBELL, 'Roy. Soc. Proc.,' A, vol. 79, p. 428.

[†] Paper read before the British Association, 1890.

brought about by using a brush consisting of a single wire perforated by a channel through which a constant flow of mercury was maintained. With regard to the variation of speed, Prof. JONES was of opinion that "no time or trouble spent in securing a constant speed will be lost for the purpose in view." It was suggested that the standard coil should consist of a single layer of wire, as the mutual inductance of the coils and disc could then be calculated with great accuracy.

The apparatus made by Messrs. Nalder Bros.* for Prof. CALLENDAR, who was then at the McGill University, Montreal, was, in general arrangement and dimensions, similar to Prof. JONES's Cardiff apparatus. Contact with the edge of the disc was made by three small tangential phosphor-bronze tubes lightly pressed on it, at points separated by angular distances of 120 degrees, and through these tubes mercury flowed on to the edge of the disc. The employment of three brushes was suggested by ROWLAND to eliminate small errors due to imperfect centering of the coil and disc, and was a distinct improvement. The possible sources of error were considered, and in a paper by Prof. AYRTON and Prof. JONES* an equation is given showing the rates of variation of the mutual inductance of the coil and disc with changes (1) in the radius A of the coil, (2) in the radius a of the disc, and (3) in the axial length $2x$ of the helix. The equation given is

$$\frac{dM}{M} = 1{\cdot}246\,\frac{dA}{A} + 2{\cdot}346\,\frac{da}{a} - 0{\cdot}0997\,\frac{dx}{x}.$$

The value of M was 45814·5 cm., of A 26·7039 cm., and of a 16·5354 cm., from which it is readily calculated that an error of 0·002 cm. in the measurement of the diameter of the disc introduced an error of nearly 5 parts in 100,000 in the value of M. An error in the measurement of the diameter of the coil of 0·002 cm. introduced an error of 14 parts in 100,000 in the value of M.

It will be seen that the experimental difficulties experienced by observers using the Lorenz method were mainly due to thermo-electric troubles at the brush contacts, want of uniformity of speed, and the effect of terrestrial magnetism. There are no difficulties attendant on the accurate evaluation of the mutual inductance of a coil and disc if the dimensions are accurately known. If a single layer coil is used, its dimensions may be determined with great accuracy; if a coil of many layers is employed, it appears best to follow Lord RAYLEIGH's practice and use two coils at a considerable distance apart in order that errors of measurement of the radius of a coil may be rendered negligibly small.

The Lorenz apparatus described in this paper was designed in 1908, our object being to determine a resistance in absolute measure with a precision within a few parts in a hundred thousand. The apparatus described eliminates the effect of terrestrial magnetism and largely reduces thermo-electric troubles at the brush contacts by the employment of two discs of equal diameters. The coils are of one layer only and

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* 'British Association Reports,' 1897.

their dimensions, together with those of the discs, may be determined with ease and with precision; in addition, any possible error in the measurement of the diameters of the discs is made negligibly small. Instead of employing one rotating conductor, a number are used, and the induced voltages may be placed in parallel or in series. The machine is of the best construction, great attention being paid to lubrication in order to avoid irregularities in the speed. As already stated, the heavy metal work was kindly undertaken by Sir W. G. Armstrong, Whitworth & Co., but the smaller work such as the chronograph, commutators, brushes, coil fittings, &c., many of which demanded great skill in their construction, were made by Mr. F. H. Murfitt, who has charge of the instrument shop of the Laboratory, and whom we also thank for many suggestions.

Section 3.—General Description.

The instrument consists of two metallic discs which support ten conducting wires rotating in a magnetic field produced by a current in four coils; an electric motor is used as the source of power. Phosphor-bronze wire brushes make contact with segments made of the same alloy attached to the ends of the rotating wires, and the difference of potential between the brush contacts at the ends of a single wire, or that between five wires, is balanced against the difference of potential between two points on a standard resistance, the current through which is the same as that flowing through the four coils. A diagrammatic sketch of the arrangement is shown in fig. 1.

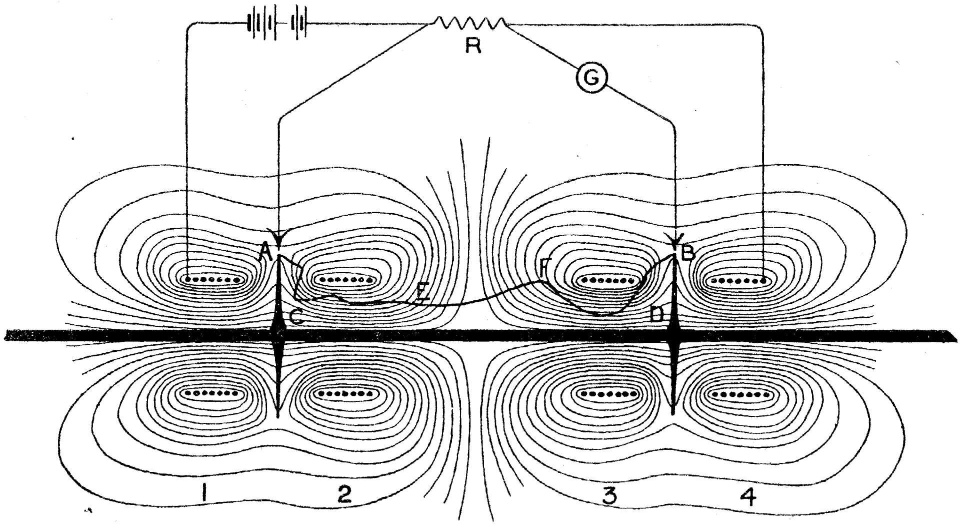

Fig. 1.

It will be seen that the current in the coils 1 and 2 is in the opposite direction to that in the coils 3 and 4. The resulting magnetic fields are opposed in direction, and the value of the field at points in the neighbourhood of the edge of a disc is zero or nearly so. A difference of potential is produced between the ends of a rotating conductor, and its value is dependent on the position only of the ends of the conductor and not upon its shape, conditionally that the conductor passes through the coils carrying the current. Thus the difference of potential at the extremities of a conductor ACDB is not altered if its shape is changed to AEFB. The segments which form the ends of the conductors are insulated from each other and from the discs.

The brushes consist of phosphor-bronze wires stretched by two spiral springs, and resemble violin bows. Each brush makes contact with one or two segments over a length varying from 5 to 6 cm., and leaves a segment at a tangent thus making the pressure greatest at the mid-point of contact. Petrol is employed as a lubricant.

There are two principal ways of using the apparatus. In the first the ten brushes are included in a circuit so as to be in series. When each brush is in contact with a single segment, the differences of potential due to five rotating conductors are added together, the remaining five conductors being ineffective. When each brush connects two neighbouring segments the ten rotating conductors are arranged in five sets of two in parallel and the total potential difference is the same as before. It is easily seen that by having a comparatively large arc of contact between each brush and a segment (or segments) and twice as many segments as brushes, the circuit made through the brush contacts is never broken.

In the second method, the brushes are divided into two sets of five in parallel and the total potential difference is the same as that of a single rotating conductor.

All the coils are wound with bare copper wire on hollow marble cylinders having double-threaded screw grooves cut on the surfaces. The two wires on any one cylinder form two adjacent helices which may be connected in series or in parallel. In the general use of the instrument they were connected in series, but they may at any time be disconnected from each other and an insulation test made between them. There are eight helices in all and these are connected by means of small concentric cables to a plug board and commutators which enable the direction of the current in any coil to be changed at will.

Each of the cylinders is mounted on a strong metal support and its position with regard to a disc may be altered with ease by screw adjustments. The distance between the mid-planes of two coils is measured by means of microscopes.

The electric motor used for driving is situated at a considerable distance from any one of the coils, and its influence on the result was calculated and also experimentally proved to be negligibly small. A commutator is fixed to the axle of the motor, and this serves to charge and discharge a condenser placed in one arm of a Wheatstone bridge. By keeping the bridge permanently balanced, the speed of the Lorenz apparatus is maintained constant. A directly driven chronograph enables the speed to be calculated.

The whole instrument is supported on gun-metal rails embedded in a solid block made with Keene's cement, sand, and ballast. With the exception of the motor, no magnetic material is used in the construction of the apparatus or supports. The complete apparatus is diagrammatically shown in fig. 2; fig. 3 is a photograph of the instrument taken from the end farther from the motor. The overall length is seven metres.

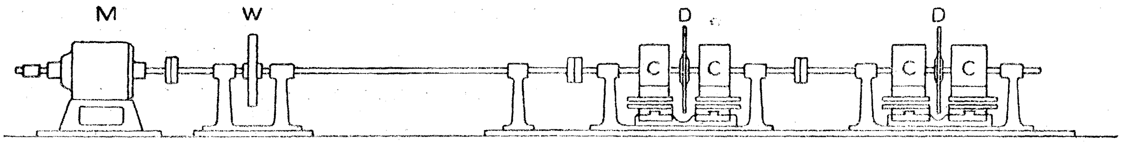

Fig. 2.

M, motor; W, fly-wheel; DD, discs; CCCC, coils on cylinders.

SECTION 4.—THE MOTOR AND FLY-WHEEL.

The electric motor M, fig. 2, is coupled to a shaft supporting a fly-wheel W; the fly-wheel shaft is in turn coupled to another supporting one of the discs, and the latter shaft is coupled to a similar one supporting the other disc. The coupling between the fly-wheel shaft and that supporting one of the discs is an insulating one, washers and tubes of stabilit being used to secure good insulation.

The electric motor is a shunt-wound one supplied by Messrs Crompton & Co., Ltd. The machine has four poles and has a commutator of the radial type. The shaft is of phosphor bronze and the motor is mounted on a base of the same alloy in order to reduce the quantity of magnetic material to a minimum. The stray field of the motor in an axial direction is surprisingly small, its intensity at a distance of 400 cm. amounting to 0·0006 C.G.S. units only. Our original intention was to enclose the motor in a double shell of soft iron, and the base of such a shell is interposed between the motor and its support; the results of our observations on the stray field showed this to be unnecessary and the idea was abandoned. The effect of the motor on the mutual inductance of the coils and discs is discussed in Section 19.

The fly-wheel is of phosphor bronze. The outer diameter is 50 cm. and the weight of metal in the rim is about 80 kgr.

SECTION 5.—THE ROTATING DISCS AND THEIR SUPPORTS.

The portion to the right of the instrument (fig. 2) consists of two similar parts, and only one of these will be described. Fig. 4 shows one of the parts.

The disc D is of rolled phosphor bronze fitted on a shaft S made of an alloy of copper and aluminium (10 per cent. aluminium and 90 per cent. copper). The original intention was to have the shaft of phosphor bronze, and two such shafts were made but were rejected on account of their appreciable magnetic susceptibility. Phosphor-bronze billets having the requisite magnetic properties could readily be obtained, but in such cases they failed to give satisfactory results with the mechanical tests and were therefore unsuited for shafting. The diameter of a disc is about 53 cm., and that of the shaft is 5 cm.

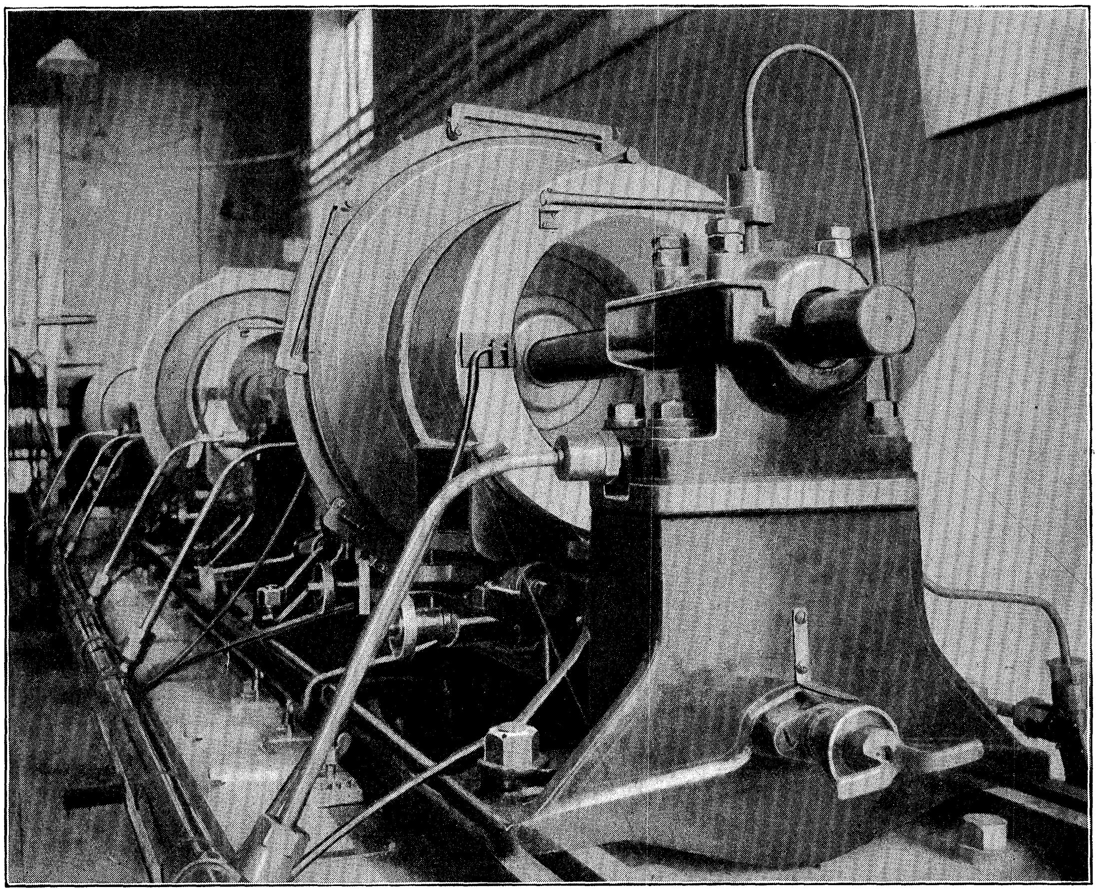

Fig. 3.

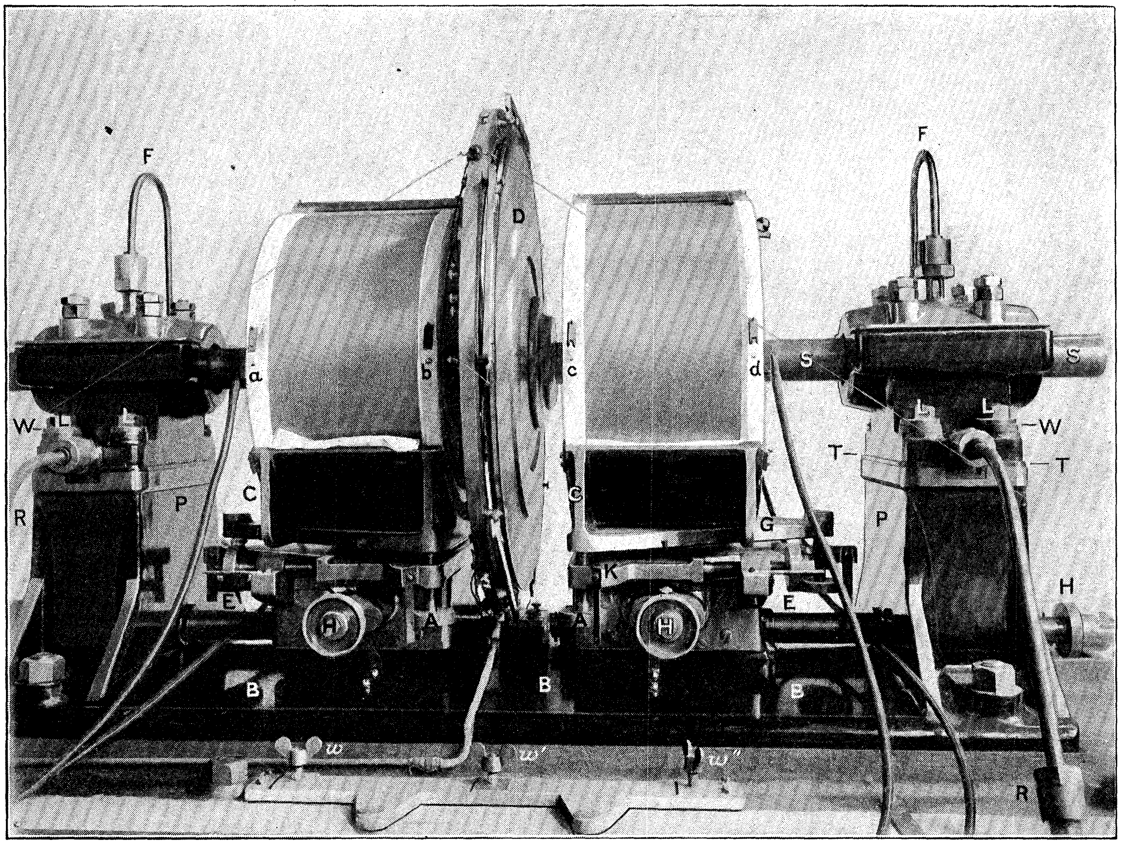

Fig. 4.

The shaft runs in bearings made of a special alloy of tin (69 per cent.), zinc (29·6 per cent.), and copper (1·4 per cent.), this being the best non-magnetic alloy we know of in which a copper aluminium shaft will run without trouble. The pedestals PP and bed B, supporting the bearings, are of phosphor bronze, except for slabs of stabilit T, 6 mm. in thickness, which insulate the bearings from the bed. The upper part of a pedestal is bolted to the lower part by phosphor-bronze bolts L, stabilit washers W, and stabilit tubes being used to insulate the upper and lower parts. The insulation resistance between the shaft and the bed has been measured on many occasions, but has never been less than a thousand megohms. If two metallic discs with uninsulated rims and conductors are employed, such insulation is essential, but it is not necessary with insulated conductors and segments such as we have used.

The oil feed (F) and return pipes (R) are of copper and are insulated from the pedestal by glass and ebonite tubes and oil-resisting rubber washers. Oil throwers are fitted to the shaft and effectually prevent the oil travelling outwards from the bearings.

SECTION 6.—THE ROTATING CONDUCTORS WHICH PASS FROM THE EDGE OF ONE DISC TO THE EDGE OF THE OTHER DISC.

We have already briefly described the system of ten conducting wires passing from disc to disc, and it will be realised that the discs in the present apparatus serve only to support the radial conductors and are employed for no other purpose. The wires used consist of No. 26 double silk-covered copper wire, shellacked, and covered with silk tube.

In order to place the ten wires in position, the shaft was drilled centrally and parallel to its length from the coupling between the two discs to points within 20 cm. of the discs, and radial holes were drilled in the shaft in these latter positions. On the coupling between the discs blocks of ebonite are screwed, and these support terminals to which the ends of the wires can be attached. Ten wires pass from the segments attached to the edge of one disc, through a channel milled in the side of the disc, and again through a brass tube screwed on the shaft, into a radial hole; after passing through the central hole drilled in the shaft they emerge at the coupling, and the free ends of the wires are attached to terminals on the ebonite blocks. Similarly, ten wires pass from the edge of the other disc to corresponding terminals, and by making suitable connections between the terminals we obtain a system of ten conductors which pass between the segments on the two discs. As already explained, the path of the conductors is unimportant, conditionally that they pass through the coils.

On the edge of a disc ten segments of stabilit are screwed. Some of these can be seen in the photograph (fig. 3), and in the sketch (fig. 5); a sectional view of a segment attached to a disc is shown in fig. 5*c*. On emerging from the channel milled in the disc, the wires pass along a groove G, milled in the stabilit, and are soldered to ten phosphor-bronze wire segments, of square section, which are screwed to the stabilit in the manner shown in the illustration. This method was adopted because of the strain to which the segments are subjected when the machine is running at full speed and which results in an increase in the effective diameter of a disc, measurements of which are given in Section 16. The wire for the phosphor-bronze segments is 25 sq. mm. in cross section and was kindly drawn for us by the London Electric Wire Company. Some of it was further drawn down into circular section wire of 0·12 mm. diameter, and this latter was employed for the brushes. Phosphor bronze was chosen for the segments and brushes because of the negligible tendency to bind and tear when this alloy is employed for moving parts in contact.

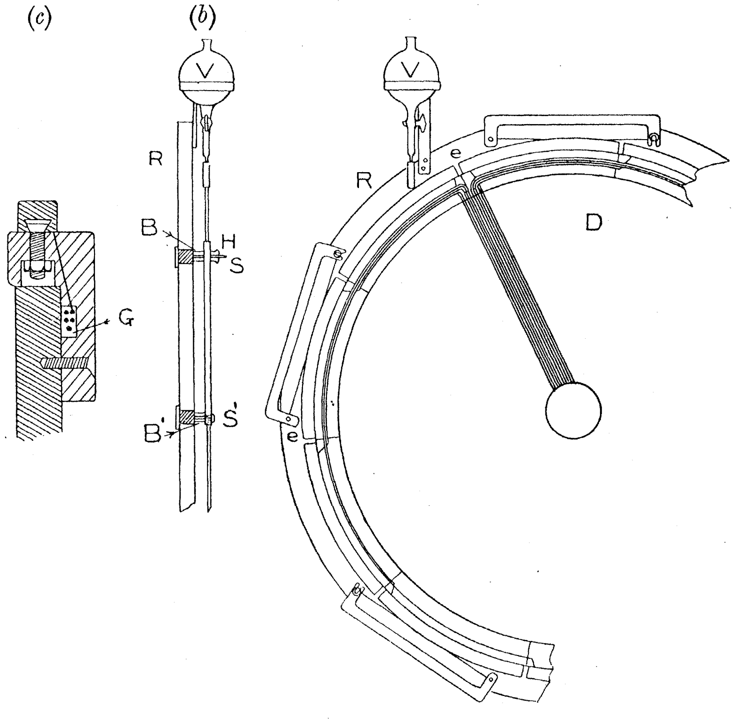

Fig. 5.

Fig. 5 shows five segments in position on a disc and illustrates the manner in which the wires pass from the segments to the main shaft. Before making any resistance measurements the shaft was rotated slowly, and with the aid of a small portable milling machine a smooth surface was turned on the segments. The sharp edges which resulted at *e*, *e* (fig. 5) were rounded in order to avoid a cutting action on the wire brushes.

SECTION 7.—THE BRUSHES AND THEIR LUBRICATION.

To obtain experience regarding the best form and number of brushes, preliminary apparatus consisting of two rotating discs of phosphor bronze was constructed in 1908.

The apparatus was astatic and our experiments were directed to eliminating trouble due to thermo-electric effects at the brush contacts.

The first brushes experimented with were of fine phosphor-bronze wire, each brush consisting of about 50 wires bound together after the manner of a common pencil brush. A light spiral spring ensured contact between a brush and the edge of a disc. A circuit through brushes, discs, and a galvanometer was completed, and the discs (25 cm. in diameter) were run at a speed of about 1500 revolutions per minute. Whatever thermo-electric effects existed at the points of contact of the brushes on one disc must have been in opposition to those at the brush contacts on the other disc, but the galvanometer deflection was far from steady, the variations in the thermo-electric effects amounting to about 0·0001 volt. Although these preliminary experiments of ours were so unsatisfactory, the results were much better than those obtained when the circuit was completed through brush contacts at the edge of one disc and others on the shaft of the apparatus.

An even more disturbing feature of our experiments was the inconstancy of the resistance of the galvanometer circuit. With one brush on each disc the resistance of the contacts was measured as something less than 0·1 ohm when the discs were stationary, but when the discs were rotating the contact resistance increased to 5, 10 and 20 ohms, and at times the circuit appeared to be broken. This effect was undoubtedly due to a vibration of the brushes brought about by the motion of the air at the edges of the discs. In practice it was not possible to prevent this by increasing the pressure applied to a brush, as by doing so the discs were worn away very rapidly and the points of contact became very hot. Shielding devices were tried with some small success, but the most favourable results were obtained by employing a number of brushes in parallel and placing in series with each brush a resistance of 10 ohms, thus ensuring that any increase or diminution of the resistance of one brush contact would have but little effect on the circuit as a whole. Our experimental results showed clearly that the employment of a large number of brushes was beneficial, but an increase in the number did not greatly improve the results, unless a resistance was placed in series with each before the brushes associated with any one disc were placed in parallel.

A further advance was made by lubricating the edges of the discs. Fatty oils and greases are impossible for such a purpose as the contact resistances are enormously increased and the general result is far worse than when no lubricant at all is used. Acheson graphite, aquadag and oildag were tried, but the results were not satisfactory. It is not very well known that paraffin oil improves all ordinary contacts such as those associated with slide wires, plugs, &c., but throughout the National Physical Laboratory paraffin oil is largely used for such purposes. We tried it therefore as a lubricant for the brushes. There was a marked improvement in the results, and we continued to use it in this preliminary work. Many kinds of brushes were tried: some were wires of phosphor bronze, some of phosphor-bronze gauze, some of copper wire, and solid slipper brushes of phosphor bronze were also used. The most satisfactory results were obtained with brushes of fine wire.

After the erection of the Lorenz apparatus we continued our experiments with brushes. The discs of the new apparatus are about 53 cm. in diameter and they revolve about 1050 times per minute, so that the velocity at the edge of a disc is about 1750 m. or 1 mile per minute. This velocity is greater than the velocity at the edges of the discs in our preliminary apparatus, and we found the brush difficulties correspondingly increased. The variation in the resistance of the galvanometer circuit due to the varying contact resistances rendered accurate work impossible, and we were led to design a brush which would not be set in vibration except to a very small extent and by the use of which thermo-electric effects would produce comparatively small disturbances. A short description of these brushes appeared in the 'Annual Report of the National Physical Laboratory for 1911,' and we are aware that the short description there published has led several investigators to try similar brushes, and the results have been reported as satisfactory.

We were guided in the design by the following considerations: (1) a fine cylindrical wire if stretched offers little resistance to a stream of air, and whatever vibration is set up will probably be regular and can be controlled by adjustment of the tension on the wire; (2) if a wire such as AB (fig. 6) is in tension and makes contact with the edge of a rotating disc D over the arc ECF, the pressure will be greatest at a point close to C the mid-point, and will gradually fall in value until at E and F it is zero. The maximum rise of temperature and the maximum thermo-electric effect will probably be at C, and the thermo-electric force will gradually diminish as we approach E and F and be zero at these points. Without the completion of any other circuit it is clear that electric currents will be produced which will flow from the wire to the disc in some parts and from the disc to the wire in other parts. If, therefore, a galvanometer is included in the circuit containing the junction of wire and disc, the resultant deflection will be very much less than that which would be produced by the *maximum* thermo-electric effect. A number of stretched wires in parallel should, of course, give better results than one. The form of brush actually used in our experiments is depicted in fig. 6. The size was largely governed by the arc of contact desired and the number of segments on the discs.

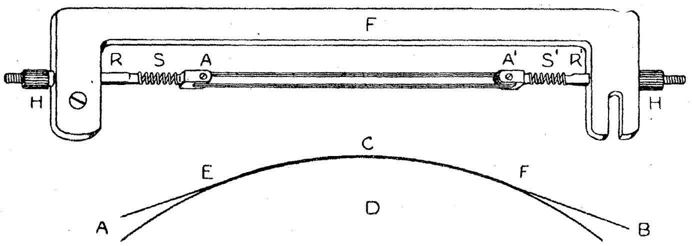

Fig. 6.

The brushes are of phosphor-bronze wire 0·12 mm. in diameter, and to obtain as small a thermo-electric effect at the contacts as possible the wire was drawn from other wire of square section similar to that employed for the segments on the discs. To make a brush, the fine wire is wound in screw cut grooves of 0·25 mm. pitch, cut on small brass cylinders which can rotate about, or be clamped to, the axles AA′. The ends of the wire are soldered to the cylinders and the wire brush thus formed is put in tension by operating the milled heads HH. The spiral springs SS′ are of stout phosphor-bronze wire and are soldered to brass rods RR′, of square section, which slide in square cut grooves cut in the frame F. The overall length of a brush is 20 cm. In practice we found the tension required for good working to be very small; a tension corresponding to the pull produced by the suspension of a 200 gr. weight was found to be most satisfactory. Some of the brushes were made with eight turns of wire, some with three turns, and a few with one turn; the majority were of three turns.

From the moment we commenced to use this style of brush with petrol as a lubricant, the thermo-electric variations produced very much less trouble. The variations are not more than one-fiftieth of those found with any other form of brush we have experimented with, and the variation in the resistance of the contacts when ten brushes are placed in series is so small that the sensitiveness of the galvanometer remains constant within the limits of error of our measurements. However, in the case of ten brushes in series, the tension on the wires has been somewhat greater than that recorded above.

Without a lubricant the brushes are not satisfactory. The wire is gripped by the disc and under certain conditions the brushes vibrate longitudinally and produce at the same time a chattering noise. Also the wire and disc become rugged and the temperature at the point of contact is very much greater than when petrol is used as a lubricant. We believe the petrol to have three beneficial influences: (1) as a lubricant; (2) as a cooling agent; (3) as a cleanser getting rid of all traces of grease and dirt. The amount of petrol to be supplied and the manner of supply was the subject of many experiments, but it is sufficient to state here the manner eventually adopted.

Fig. 5 shows a brush in position and a petrol supply vessel V. The brush is secured to a massive phosphor-bronze ring R (see figs. 3 and 5) by the screws SS′, and it can be placed in contact with a segment or removed from such contact by loosening the terminal head H and giving the brush frame a circular motion about the pivot screw S′. The screws SS′ and the brass distance pieces BB′ are insulated from the ring R by means of ebonite sleeves, the latter allowing of some adjustment of the screws SS′. The ring R is supported by the bed B (fig. 4) but is insulated from it. Stretched phosphor-bronze wires support the ring laterally and give it the necessary rigidity.

The petrol supply vessel is of glass, the tube being drawn down to a capillary about 0·5 mm. in diameter at its lower end. On the narrow glass tube thus formed, a piece of circular lamp wick about 3 cm. long is slipped; the lower end of this wick just touches the rotating segments and thus feeds petrol directly on to the surface of the segments. The usual rate of supply of petrol was about 500 c.c. in twenty minutes. When the apparatus is running well, an observer may place the end of a finger on the rotating segments and find it well flooded with petrol removed from the rim. The wicks are renewed at least every day and the brushes wiped with clean chamois leather. In our experiments a set of ten brushes lasted usually for six or nine complete sets of observations; after that number the surfaces of the wires became somewhat rough and the results were not quite so satisfactory. The magnitude of the changes of the thermo-electric effects with two sets of five brushes in parallel will be realized when it is said that often for intervals of twenty minutes the rapid variations in the total thermo-electric voltage did not exceed 0·1 microvolt. A slow progressive variation was commonly observed, but this was not a source of trouble.

SECTION 8.—THE COIL SUPPORTS.

Each marble cylinder weighs about 50 kgr. and is supported on a phosphor-bronze cradle C (fig. 4) so that its axis is coincident with the axis of the shaft. The base of the support is a triangular casting G, which is supported in turn by three levelling screws A on the "hole, slot, and plane" principle. The pitch of the levelling screws is one millimetre and the heads of the screws are divided into one hundred equal divisions, thus enabling any particular vertical motion to be repeated within one-hundredth of a millimetre.

A second triangular casting K carries the levelling screws and is fitted over a large central stud attached to a slide, so that the cylinder may be rotated about a vertical axis if necessary. The maximum angular motion is 12 degrees and the magnitude of any motion can be directly read on the engraved head of the horizontal screw E to half a minute of arc. Backlash is avoided by the use of strong phosphor-bronze springs.

A cylinder and the cradle supporting it can be moved 5 cm. in two horizontal directions at right angles by means of two slides, the motions being controlled by screws, the heads H of which are divided to read hundredths of a millimetre as in the case of the screws for the vertical motion. For these movements also strong phosphor-bronze springs are employed to prevent backlash.

SECTION 9.—LUBRICATION OF BEARINGS.

The bearings of the machine are nine in number, each of those of the motor being about 9 cm. long and each of the remaining seven about 14·5 cm. long. The lubricant used is best turbine oil which is fed to the bearings under a pressure of about 15 lbs. per square inch, the rate of supply being a cubic foot of oil every five minutes. The oil supply tank and pump are of phosphor bronze and are situated under the floor, $1\frac{1}{4}$ m. below the level of the bearings of the main shaft, and 4 m. distant from the nearest point of the machine. The pump is driven by a small motor of one-eighth horse-power and forces the oil through copper pipes to the bearings of the machine. After passing through the bearings the oil returns through copper drain pipes to the tank and is strained through fine copper gauze before again entering the pump. The system works excellently, no trouble whatever having been experienced.

SECTION 10.—REGULATION OF SPEED.

On the axle of the motor a commutator is fitted which serves to charge and discharge a condenser four times for each revolution of the shaft. The condenser is placed in one arm of a Wheatstone bridge, the other arms of which are platinum-silver resistance coils. Balance of the bridge results for a particular frequency only of charge and discharge, and to maintain a balance over a considerable length of time the frequency and therefore the speed of the Lorenz apparatus must be kept constant. To ensure constancy of the arms of the bridge, the condenser and the platinum-silver resistances were kept in a constant temperature room ; a small variable resistance in series with one of the arms was in general adjusted to secure a balance when the speed was that best suited for the resistance measurements, but after the latter measurements had commenced the speed only was controlled to maintain the balance. The galvanometer used was a suspended coil instrument, the spot from which was received on a ground glass scale mounted over the fly-wheel ; an assistant observer, S. WATTS, applied a variable pressure to the fly-wheel and so maintained the balance of the bridge. A change in the speed of the Lorenz apparatus of 1 part in 10,000 produced a deflection of the galvanometer spot of 4 mm., and in general, a balance was maintained for twenty minutes or more with a maximum deflection not greater than 2 mm. ; occasionally, better results than this were obtained. An adjustable resistance is in series with the field coils of the motor, and before attempting to govern the speed this resistance is altered until a speed results which is very slightly greater than that desired.

The motor was run from a battery of large storage cells and very good results were obtained with no regulation at all, if the current flowed sufficiently long (generally from one to two hours) to raise the field coils to an approximately constant temperature. However, the method finally adopted enabled resistance measurements to be made a few minutes after starting the motor. A somewhat similar method of controlling speed for measurements of capacity has been in use for several years both at the National Physical Laboratory and at the Bureau of Standards.

SECTION 11.—METHOD OF RECORDING THE SPEED.

The chronograph described is very similar to one designed at the Bureau of Standards,* and made by the Société Genevoise, for details of which the writer

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* 'Bull. Bureau of Standards,' vol. 3, p. 561.

is indebted to Dr. ROSA and other members of the Bureau of Standards. The instrument described below, which is of brass, was made by Mr. MURFITT of the Instrument Department of the Laboratory.

The method of recording the speed is a direct one. A chronograph drum is geared to the main shaft of the apparatus through the medium of two worm wheels and two spur wheels, the gear ratio being 174. The usual speed of the main apparatus is about 1044 per minute, and under such conditions the drum of the chronograph makes one revolution in ten seconds. An electro-magnet is supported on a carriage which is connected to a split-nut engaging with a screw of 1 mm. pitch. The direction of travel of the carriage is parallel to the axis of the drum and it advances 1 mm. for each revolution of the drum. Every second a current passes through the electro-magnet and the latter operates a small punch, which, striking through a typewriting ribbon, prints a dot on a sheet of paper carried by the drum. The circumference of the drum is exactly 500 mm., so that when six revolutions per minute are made, successively recorded dots are 50 mm. apart. The split-nut fixed to the carriage carrying the electro-magnet can be disengaged from the driving screw and the carriage rapidly run along the rails supporting it. When the printing punch is pressed forward during this operation a line parallel to the direction of travel of the electro-magnet is printed on the paper. This line is hereafter called the base line and by measuring the angle between it and a row of dots recorded during a run, the speed can be calculated with great accuracy.

The method of calculating the speed is as follows: At a speed of exactly 1044 revolutions per minute there are $1044/174 = 6{\cdot}0000$ revolutions of the chronograph drum per minute. The resulting record will therefore be ten rows of dots, the dots being 1 mm. apart, and the rows 50 mm. apart and parallel to the base line. If the speed is slightly diminished or increased, the rows of dots will slope upwards or downwards. In practice, a set of observations for the measurement of a resistance is made to last for at least 1000 seconds (*i.e.*, about 17 minutes), and the resulting rows of dots are therefore more than 10 cm. long. From a portion of the base line lying beneath (or above) a row of dots a length of 10 cm. is marked off, and from the extremities of the 10 cm. line ordinates are drawn to the nearest row of dots. If these ordinates are equal in length the rows are parallel to the base line, and the distance between two dots recording an interval of 1000 seconds (10 cm. run) is 50,000 mm. if the distance is measured along the trail of the recorded seconds. If the ordinates are not equal in length the machine was running either faster or slower than 1044 revolutions per minute; which of these holds good is decided by the direction of the slope. Suppose the difference between the ordinates is 14·5 mm. and that the machine was running faster than 1044 per minute. The mean speed is calculated to be $1044 \times \frac{50{,}000}{50{,}000-14{\cdot}5} = 1044{\cdot}30_3$ revolutions per minute. It is clear that a difference between the ordinates of half a millimetre corresponds to 1 part in 100,000 of the speed. The drum is 50 cm. long and a record lasting 80 minutes can therefore be made.

Fig. 7 is a full-size reproduction of a portion of a record, taken June 26, 1913. The recorded speed is $1043{\cdot}96_2$ revolutions per minute. The rows of dots enable the mean speed to be calculated with an error certainly not greater than 1 part in 100,000, and the speed throughout (as illustrated by the rows of dots lying in a straight line and not a wavy one) is wonderfully uniform. The general fluctuations of speed cannot be detected on the record, but their magnitude has been estimated in another way as explained in Section 23.

A mercury contact is used on the pendulum of the standard clock and a relay is operated continuously by the current which passes. The rate of the clock is of course allowed for.

SECTION 12.—MAGNETIC TESTS.

Magnetic tests were made on all the materials employed. With the exception of the motor, we are satisfied that the permeability of no part of the machine and the concrete bed on which it rests differs from unity by more than 2 parts in 100,000.

Every part of the machine bears a distinctive number. When each part was cast, a lug about 5 cm. long and 5 cm. in diameter was cast with it, and after being stamped with the same number as the casting it was forwarded by Sir W. G. Armstrong, Whitworth & Co. to be tested for magnetic quality. In the case of springs, rods, tubes, &c., pieces were cut from these and used for the tests.

The method of testing was similar to that employed for the parts of the current balance.* Soft iron wire and ferrous sulphate were used to calibrate the apparatus and sufficient sensitiveness was obtained to detect a difference from unit permeability of about 1 part in 100,000. Thus, when powdered ferrous sulphate having a permeability of about 1·00044 was contained in a glass tube having the same cross section as most of the test pieces, the resulting galvanometer deflection was 5·2 mm. The test pieces of marble were of much larger cross section than the glass tube and the sensitiveness was correspondingly increased.

Trouble was experienced with a number of brass rods and tubes and with the shaft of the apparatus. The first shaft was made of phosphor bronze and had a magnetic permeability of 1·006; in consequence this shaft was rejected. The material obtained for the second shaft was of an alloy of copper and aluminium (copper 90 per cent., aluminium 10 per cent.) and was also rejected as its permeability was about 1·002. The third shaft was made with specially pure aluminium and copper in the same proportions as before, and its permeability differs from that of air by an amount too small to be detected. The billet for the shaft was prepared by The Broughton Copper Company, Limited, of Manchester, and we thank them for the care taken in its preparation.

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* 'Phil. Trans,' A, vol. 207, p. 475, 1908.

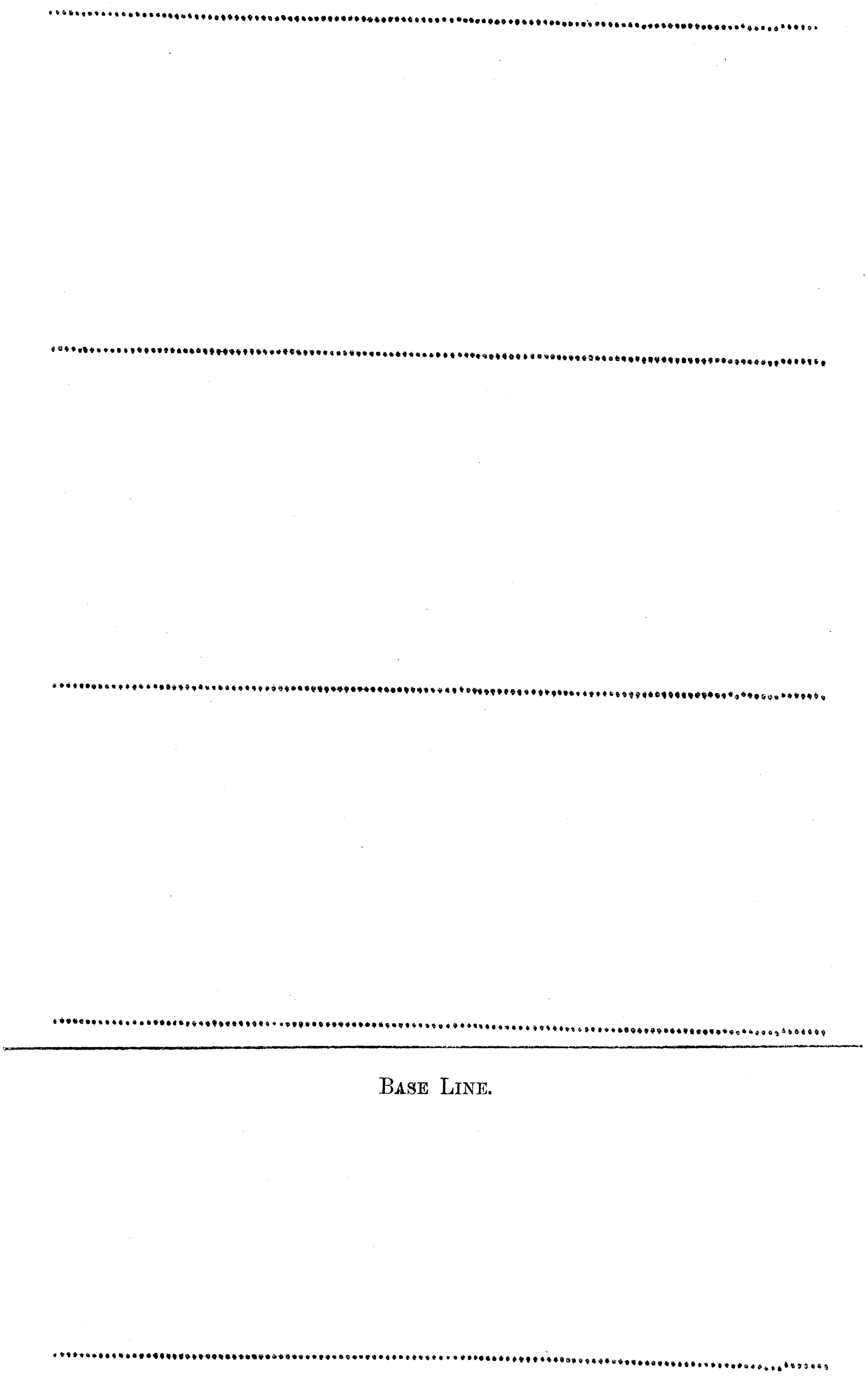

Fig. 7.

The samples of Portland cement tested varied in magnetic permeability from 1·0005 to 1·0020. It is possible that metallic iron finds its way into the cement from the steel grinding machines, and the Associated Portland Cement Manufacturers very kindly offered to exclude such contamination by grinding some cement through millstones. At the same time the manufacturers pointed out that a certain amount of oxide of iron in combination with lime and silica is invariably present in Portland cement, and Keene's cement was suggested as being suitable for our purpose. This is a white cement absolutely free from iron compounds of any sort. We tested several samples for magnetic quality, and finding it quite satisfactory decided to use it for the block on which the apparatus rests. No trouble was experienced in obtaining sand and ballast free from magnetic substances.

SECTION 13.—CONSTRUCTION AND MEASUREMENT OF THE COILS.

"First Statuary" Carrara marble was chosen for the material of the cylinders. We were guided in our choice by our experience with the cylinders for the Ayrton-Jones current balance, the tests then made showing marble to be an excellent electrical insulator and of negligible magnetic susceptibility.

The cylinders were prepared in the rough by Messrs Walton, Gooddy & Cripps, whom we thank for the trouble they took in choosing masses of marble practically free from veins. The small shiny specks which are often present in Carrara marble consist of iron pyrites which has a magnetic susceptibility of about 0·0005. The conductivity of iron pyrites is much greater than that of marble and any small specks on the surface of a cylinder should therefore be removed; we found that a small crystal of pyrites pressed between two plates of copper reduced the insulation between the plates from a value which was practically infinity to 1000 ohms. Clearly a large number of crystals at the surface of a marble cylinder would introduce a serious source of error.

The coefficient of expansion of the marble was determined by direct measurements on the cylinders and found to be $5·0 \times 10^{-6}$ for 1° C., the temperature range being from 11° C. to 20° C. These measurements are described in the section dealing with the measurements of the mean diameters of the coils.

The cylinders chosen are free from flaws and cavities. Of six cylinders which were submitted, two exhibited "ground flaws" and were in consequence rejected. These "ground flaws" appear to have been produced by a rupture in the marble many thousands of years ago and subsequent re-union by pressure or equivalent agency. In all, Messrs Walton, Gooddy & Cripps prepared 16 cylinders in the rough and the four best of them were chosen for our work.

The marble cylinders for the Ayrton-Jones current balance were baked in an oven at 140° C. for 30 hours and afterwards immersed in hot paraffin wax. At the same time a marble rod was subjected to similar treatment, and since that time this rod has been kept under observation in the Metrology Department. Possibly there is a flaw in the rod, but it is certain its length has not kept so constant as other rods which were not baked and immersed in wax. We decided, therefore, to omit the treatment in the case of the four cylinders for the Lorenz apparatus.

The cylinders were turned in a manner very similar to that employed for the suspended coils of the current balance. The inner and end surfaces of a cylinder were turned with the cylinder fixed to the face plate of the lathe, but the outer surfaces and double spiral grooves were cut when the cylinder was mounted on a specially constructed mandrel supported between dead centres. The inner and outer surfaces of a cylinder are practically concentric and the ends at right angles to the axis.

The turning of each cylinder occupied about four weeks. The cylinders referred to as Nos. 1 and 2 were turned by the late Mr. Taylerson of the Engineering Department of the Laboratory. Mr. Taylerson had marked ability for accurate work of this kind and made many useful suggestions during the progress of the work. The turning of cylinders Nos. 3 and 4 was very ably done by Mr. Tribe of the Engineering Department.

The winding of the coils was carried out in a manner identical with that employed for the coils of the current balance. It is only necessary to state here that the coils were wound with the wire in tension, the effective load on the wire during winding being 4 kgr. The cylinders were rotated very slowly and after each revolution a stoppage was made for measurements to be taken of the diameter of the wire.

The leads of all the coils must lie in a plane containing the axis of the cylinder, for otherwise the mutual inductance of the leads and the discs will not be zero. To ensure the absence of any correction due to such a cause, the connections to the coils were made in the manner shown in fig. 8. At the points where the coils terminate, two radial cylindrical holes *a* and *b* are drilled and enlarged to admit of the slotted brass nipples NN′ being screwed into them. Soldered connections are made between the nipples NN′, the leads *c* and *d*, and the leads through *a* and *b*. The leads *c* and *d* pass to two brass blocks mounted on an ebonite piece screwed to one end of the cylinder, and to these same blocks a small concentric cable is secured in the manner illustrated in the figure. The whole of the leads shown in the figure lie in an axial plane of the coil and this same plane contains the leads connected to the other coil on the cylinder.

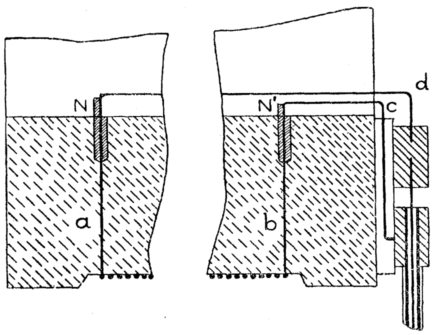

Fig. 8.

An estimate was made of the accuracy with which the number of turns is known. On each cylinder there are two coils each having 96 turns and a diameter of about 36 cm.; the length of wire to each coil is, therefore, about 108 m. From observations on the radial holes and a consideration of the methods adopted for drilling them, the number of turns is considered to be correct within 2 parts in 1,000,000.

The copper wire with which the eight coils are wound was supplied by The London Electric Wire Company on bobbins of the same diameter as the cylinders. It is hard drawn No. 24 S.W.G. and its mean diameter, obtained from about 800 measurements taken when winding the coils, is $0{\cdot}557_5$ mm. The diameter was also measured in the machine employed to determine the diameter of the coils, the mean of eighty measurements being identical with that already given.

Measurement of the Mean Diameters of the Coils.

Twelve axial planes at angular distances of 15 degrees apart are marked on the end faces and on the ungrooved portions of the outer cylindrical surfaces of each cylinder. These planes are numbered 1 to 12 and are the reference planes in the diametral measurements. Each cylinder was mounted in turn on the mandrel employed for turning the outer surfaces and supported between dead centres attached to the measuring machine.

This machine was made by Sir W. G. Armstrong, Whitworth & Co. at their Openshaw Works, to designs prepared by the firm in collaboration with Mr. L. F. RICHARDSON, formerly of the Metrology Division of the Laboratory. It consists of a straight bed carrying two fixed headstocks with cone-centres, between which the coil, on its mandrel, can be mounted. A saddle sliding on the bed carries two measuring headstocks on a slide which is adjustable so as to bring the line of centres of these headstocks exactly perpendicular to that of the fixed headstocks. Each of the measuring headstocks contains a barrel which can be moved in and out along the line of centres by means of a carefully calibrated micrometer screw ; and sliding freely in the centre of each barrel is a plunger, the front end of which constitutes the measuring face, while the rear end carries a small knife-edge pressing against a vertical lever pivoted in the barrel. At its upper end this lever carries a sensitive spirit level. The barrel is advanced by means of the micrometer screw until the plunger, being arrested by contact with the object to be measured, tilts the level far enough to bring the bubble to a definite mark. The reading is then taken with the aid of a vernier on the measuring wheel attached to the micrometer screw, to $\frac{1}{100{,}000}$ inch. The same operation is carried out with each of the two measuring headstocks, one at either end of a diameter, and the sum of the readings is compared with the sum of similar readings on a gauge bar of known length. For convenience, this gauge bar is mounted on the machine during the measurements, so that by simply traversing the saddle from the coil to the gauge the zero reading can be checked at any stage of the work. To provide for this, one of the cone centres is made specially large, and pierced behind the cone by a hole through which the gauge can be passed.

In the original design the control of the vertical lever carrying the level was effected by means of coiled springs contained in the barrel. It was found, however, that this arrangement gave rise to a greater pressure than was desirable at the point of contact between the plunger and the coil. Owing to the small radius of the wire with which the coils are wound, a comparatively slight pressure would produce deformation sufficient to lead to errors in the readings. A calculation showed that in order to keep errors due to this cause within allowable limits (less than $\frac{1}{1000}$ mm. on the diameter) the pressure must not exceed four or five ounces. At the suggestion of Mr. J. E. SEARS, the springs were taken out and the vertical lever was extended downwards and provided with a weight at its lower end, thus forming it into a pendulum with a gravity control which could be adjusted to a nicety, At the same time the form of the body of the plunger was slightly modified, and its bearing surfaces lightly polished, so as to minimize friction. In this way the contact pressure was reduced to within the required limits.

Many measurements of the coils were made with a current passing through them in order to reproduce as nearly as possible the conditions under which they would actually be used. To enable this to be done, the plungers were provided with non-conducting measuring faces of polished agate. These proved highly satisfactory in use, and the measurements made with the machine were of quite remarkable accuracy.

Two pairs of Hartmann steel gauges were employed in the measurements. The following table gives their values at 0° C. :—

TABLE II.—Giving the Values of the Gauges employed.*

Approximate length of gauge in millimetres.	Sèvres value.	N.P.L. determination, using the Sèvres value of the 1000 mm. gauge as basis.
	mm. μ	mm. μ
60	60 - 1·5	60 - 1·3
160	160 - 0·7	160 - 0·7
200	200 + 0·1	200 + 0·4
300	300 - 1·1	300 - 0·9

By addition we get

	Sèvres value.	N.P.L. value.
	mm. μ	mm. μ
Gauges 300 + 60	360 − 2·6	360 − 2·2
,, 200 + 160	360 − 0·6	360 − 0·3

The Sèvres values are stated to be accurate within 0·5μ, and the probable error of

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* μ = 1 micron = 0·001 mm.

the N.P.L. values is of the same order. By taking the means of Sèvres and N.P.L. determinations as the best obtainable, we have

$$300+\ 60 = 360\text{ mm.}-2{\cdot}4\mu\text{ at }0^\circ\text{ C.},$$

$$200+160 = 360\text{ mm.}-0{\cdot}4\mu\text{ at }0^\circ\text{ C.}$$

Using the dilatation equation determined at Sèvres on a bar of the same material

$$L_t = L_0\left\{1+(10{\cdot}469+0{\cdot}0035_2t)\,t\times 10^{-6}\right\}$$

we get the following lengths at 17° C. :—

$$\text{Gauges }(300+\ 60) = 360{\cdot}062_0\text{ mm.},$$

$$,,\quad (200+160) = 360{\cdot}064_0\text{ mm.},$$

with an increase of $3{\cdot}8\mu$ on the 360 mm. length per 1° C. increase in temperature at or about 17° C.

To obtain the mean temperature of the marble, holes were drilled in the cylinders and in these holes thermometers were inserted. The holes are parallel to the axis of the coils, and the depth of each is about half the axial length of a cylinder, so that the temperature recorded by a thermometer is very approximately the mean temperature of the cylinder.

The temperature coefficient of expansion of the marble was determined by making diametral measurements at temperatures varying from 11° C. to 20° C. Assuming that the coefficient of expansion of the steel gauges is correct, the mean coefficient of linear expansion of the marble was found to be $5{\cdot}1\times 10^{-6}$ for the range 11° C. to 20° C. The observations were remarkably consistent, as will be seen from fig. 9, in which the difference of expansion of the steel and marble is plotted against temperature. During these observations we were much struck with the comparative rapidity with which the temperature of the marble followed that of its surroundings. The temperature coefficient of expansion of the marble was also deduced from measurements of the axial length of a cylinder. In this case the difference of expansion of invar and marble was directly recorded, and taking the invar as known the value obtained for marble is $4{\cdot}9\times 10^{-6}$, which is in good agreement with the value given above. When required in our work we took the mean of these values, *i.e.*, $5{\cdot}0\times 10^{-6}$, as correct.

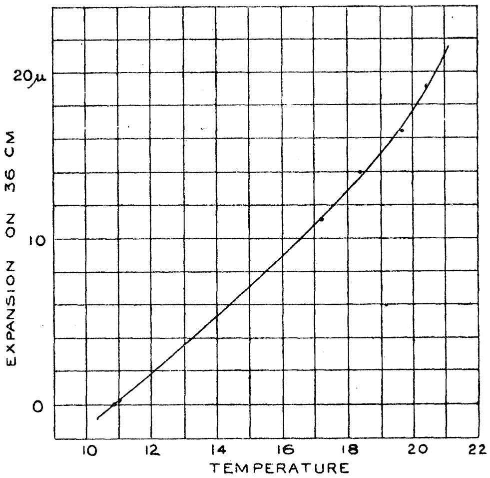

Fig. 9. Differential expansion, steel-marble.

As already stated, the contact pieces of the measuring machine are of agate, in order that measurements of diameters may be made while a current is passing through the coils. The current used in our absolute measurements of a resistance did not exceed 2 amperes, and we experimented with such a current through the coils during our measurements of the diameters. Preliminary observations showed that the expansion of a cylinder 5 minutes after switching on the current was sufficiently great to allow of a definite measurement being made, and a thermometer recorded an increase of temperature of a whole degree in this time. A survey of the surface of the coil showed the expansion to be greatest in the mean diametral plane, and least at the ends; measurements were, therefore, made on four turns of wire, these being the 1st, 16th, 48th, and 96th strands measured from one end.

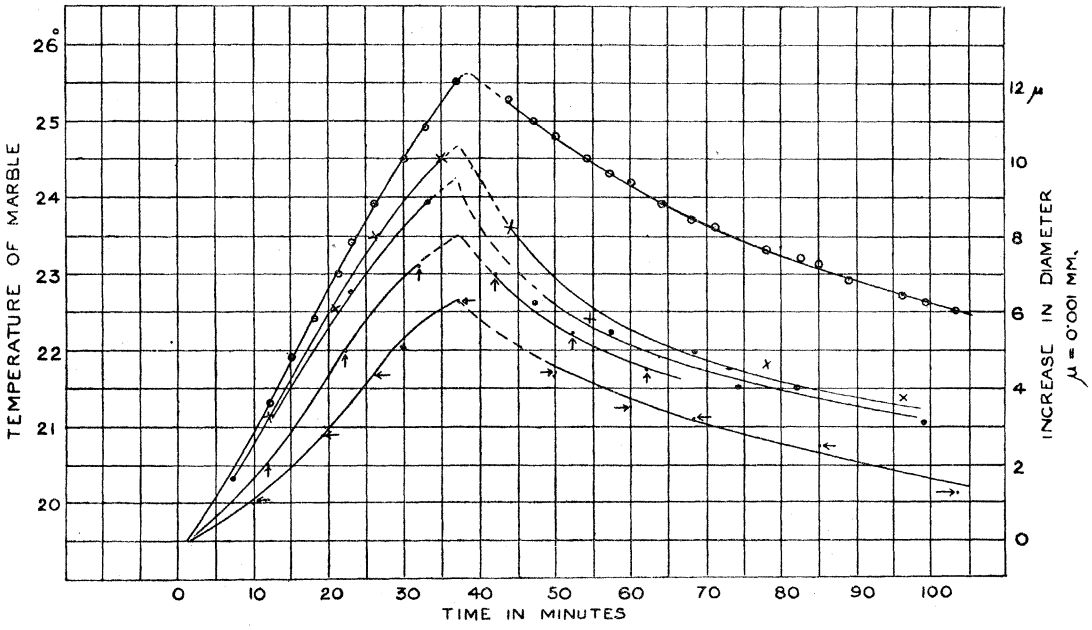

Fig. 10. Showing increase in temperature of marble cylinder and increase in diameter of certain parts of the coils when a current of 2 amperes is passed through them for 37 minutes.

⊙ indicates temperature observations
× „ observations on strand No. 96 ⎫
● „ „ „ „ 48 ⎬ measured from one end of coil.
↑ „ „ „ „ 16 ⎪
← „ „ „ „ 1 ⎭

The results of the measurements are given in figs. 10 and 11. In fig. 11 the expansion of the coil is given 5, 10, 15, 20, 25, and 35 minutes after a current of 2 amperes was switched on. The maximum increase in diameter is 10·4μ. Fig. 10 shows the rise in temperature recorded by the thermometer when the current was left on for 37 minutes, and shows also the fall in temperature after the circuit was broken. A relation between the mean rise in temperature due to the current and the change in mutual inductance of the coils and the discs was worked out and found to be extremely useful when making the resistance measurements. The relation is as follows:—"If, in the resistance measurements, t_1 is the mean of the initial temperatures of the marble cylinders, and t_2 the mean of the final temperatures, the mean mutual inductance (during the resistance observations) of all the coils and the two discs is the same as when the temperature of the cylinders is uniform throughout and equal to $t_1+0{\cdot}42\,(t_2-t_1)$ other things being kept constant."

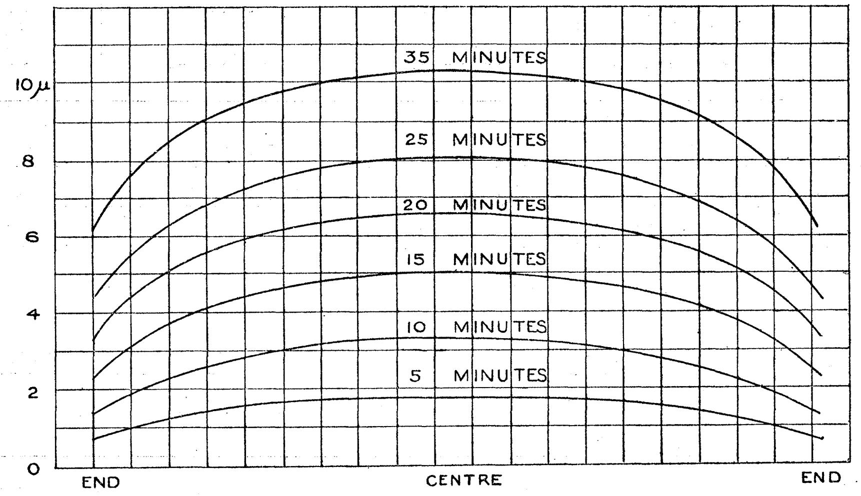

Fig. 11. Showing expansion of coils on marble cylinders at intervals from 5 to 35 minutes, when a current of 2 amperes is passed through them.

Complete measurements of the coils have been made on three occasions. The first set of measurements was made in February and March, 1912, and two sets were made in April, 1913. In the first set 192 observations were made on each coil, 16 observations being made in each of 12 axial planes 15 degrees apart. The turns of wire measured in any one plane were 1 cm. apart, and neighbouring strands were measured in succeeding planes, so that one measurement was made on every turn of wire.

In the measurements made in 1913, six measurements were made in each of 16 diametral planes 1 cm. apart, alternate sets of six measurements being made in axial planes 30 degrees apart, and the remaining measurements were made in planes midway between these. In addition, 16 observations were made in each of three axial planes 60 degrees apart, thus making 144 observations in all. Both pairs of gauge bars were used, and the difference found between them is identical within $0{\cdot}5\mu$ with that deduced from the values already given.

In the following table we give the mean diameters of the coils on the four cylinders as deduced from the measurements made in February and March, 1912, and April, 1913:—

TABLE III.—Giving the Mean Diameters of the Coils at 20°·0 C.

Coil on cylinder No.	February–March, 1912.	April, 1913.		Mean.
		(*a*)	(*b*)	
	cm.	cm.	cm.	cm.
1	$35{\cdot}8808_3$	$35{\cdot}8807_2$	$35{\cdot}8806_8$	$35{\cdot}8807_4$
2	$35{\cdot}8817_6$	$35{\cdot}8815_2$	$35{\cdot}8815_0$	$35{\cdot}8815_9$
3	$35{\cdot}8855_4$	$35{\cdot}8853_0$	$35{\cdot}8853_4$	$35{\cdot}8853_9$
4	$35{\cdot}8867_4$	$35{\cdot}8866_2$	$35{\cdot}8864_4$	$35{\cdot}8866_0$

The values recorded in columns 2 and 3 are exactly the means of 192 and 102 measurements respectively. The values in column 4 are not quite the means of the 48 observations. The cylinders are not quite circular in cross-section, and the mean of 48 measurements in three planes should, in general, be different from the mean of

TABLE IV.—Giving the Results of Measurements of the Diameters of the Coils on Cylinder No. 1. Temperature 20°·0 C.

Axial plane No. . .	1	2	3	4	5	6	7	8	9	10	11	12	
Measurements on turn No.	Diameter = 35·8750 cm. +												Mean.
	μ.	μ.	μ.	μ.	μ.	μ.	μ.	μ.	μ.	μ.	μ.	μ.	μ.
1	29		14		22		27		30		30		$25{\cdot}_3$
12		27		26		31		36		36		23	$29{\cdot}_8$
24	34		30		32		36		35		35		$33{\cdot}_7$
36		35		37		42		41		47		45	$41{\cdot}_2$
48	41		39		39		46		48		49		$43{\cdot}_7$
60		40		42		48		47		49		48	$45{\cdot}_7$
72	61		57		59		61		64		63		$60{\cdot}_8$
84		65		66		69		70		75		65	$68{\cdot}_3$
96	67		65		76		74		72		70		$70{\cdot}_7$
108		70		69		72		69		78		66	$70{\cdot}_7$
120	58		61		63		65		72		66		$64{\cdot}_2$
132		66		65		73		74		73		68	$69{\cdot}_8$
144	67		63		68		69		71		72		$68{\cdot}_3$
156		66		70		74		76		70		73	$71{\cdot}_5$
168	75		67		73		71		72		71		$71{\cdot}_5$
180		65		67		70		72		65		66	$67{\cdot}_5$
192	64		65		70		74		78		70		$70{\cdot}_2$
Mean . . . =	$55{\cdot}_1$	$53{\cdot}_9$	$51{\cdot}_2$	$55{\cdot}_2$	$55{\cdot}_8$	$59{\cdot}_9$	$58{\cdot}_1$	$60{\cdot}_6$	$60{\cdot}_8$	$61{\cdot}_6$	$58{\cdot}_4$	$56{\cdot}_8$	$57{\cdot}_2$

$\therefore$ Mean diameter (centre to centre of wires) $= 35{\cdot}8750 + 0{\cdot}0057_2 = 35{\cdot}8807_2$ cm. at 20°·0 C.

measurements made in a large number of planes. This difference can be obtained from curves showing the cross-section of the cylinders, and in all cases we have estimated this difference and applied a correction.

It will be observed that the means of the 1913 measurements are about 2μ less than those of 1912. This difference cannot be due to the gauge bars employed, and we believe it is not due to errors of observation; it is probably due to a real diminution in the diameters of the coils due to a displacement of small crystals of marble under the wires. This is quite possible, as the coils are wound under tension and have been subjected to considerable changes in temperature (due to the current passing through them) since 1912.

TABLE V.—Giving the Results of the Individual Measurements on the same Coils, when Measurements were made in Three Planes only.

Axial plane No. . .	Between 3 and 4	Between 7 and 8	Between 11 and 12	
Measurements on strand No.	Diameter = 35·8750 cm. +			Mean.
	μ.	μ.	μ.	μ.
2	24	28	31	$27{\cdot}_7$
12	24	29	32	$28{\cdot}_8$
24	26	32	35	$31{\cdot}_0$
36	32	35	38	$35{\cdot}_0$
48	40	42	47	$43{\cdot}_0$
60	42	45	47	$44{\cdot}_7$
72	54	66	65	$61{\cdot}_7$
84	61	66	67	$64{\cdot}_7$
96	58	61	69	$62{\cdot}_7$
108	67	71	74	$70{\cdot}_7$
120	57	69	63	$63{\cdot}_0$
132	61	67	67	$65{\cdot}_0$
144	56	75	70	$67{\cdot}_0$
156	71	79	76	$75{\cdot}_3$
168	67	80	74	$73{\cdot}_7$
180	68	75	71	$71{\cdot}_3$
192	70	79	75	$74{\cdot}_7$
Mean . . . =	$51{\cdot}_6$	$58{\cdot}_5$	$58{\cdot}_9$	$56{\cdot}_3$

$\therefore$ Mean diameter $= 35{\cdot}8750 + 56{\cdot}_3\mu = 35{\cdot}8806_3$ cm. at 20°·0 C.

From fig. 12 and the observations in Table IV. we conclude that the mean diameter in these three planes is less than that of the complete coils by $0{\cdot}5\mu$. Hence mean diameter of the coils $= 35{\cdot}8806_8$ cm. at 20°·0 C.

Tables IV. and V. give the results of the individual measurements made in April, 1913, of the coils on cylinder No. 1.

The curves given in fig. 12 show to what extent the coils vary in diameter. The conicality of the coils on cylinders 3 and 4 is much less marked than in the case of those on cylinders 1 and 2, but the variation from a circular cross-section is very small for all of the coils. The variation of diameter in any cross-section is so small that the mean diameter of the cross-section may be employed for the purposes of the calculation of the mutual inductance without appreciable error, but the variation in diameter from end to end of the coils necessitates the application of a conical correction.

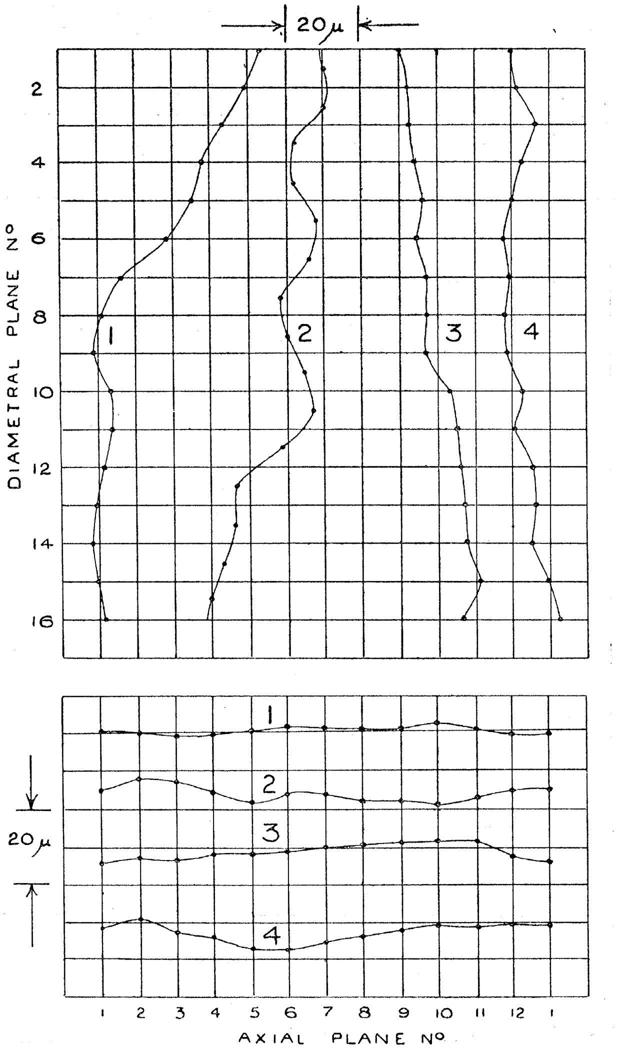

Fig. 12. Showing extent to which coils are conical, and variations in cross-section from circle.

The probable error of the mean diameter of the coils is estimated to be not greater than 1μ. The probable errors of the gauge bars used are stated on p. 54; the probable errors of the observations may be estimated from the data given in Table III.

Measurement of the Axial Lengths of the Coils and the Variation of Pitch.

The pitch measuring machine, also constructed by Sir W. G. Armstrong, Whitworth and Co., consists of a straight horizontal bed provided with dead centres on which the mandrel carrying the coil can be mounted. Parallel to the line of centres is a slide, along which a small slide rest can be traversed by means of a calibrated measuring screw, of $\frac{1}{10}$-inch pitch. The measuring wheel which actuates this screw can be read to $\frac{1}{100,000}$ inch by means of a vernier, but the accuracy of repetition of readings is somewhat less than this. The slide carries a microscope whose optical axis intersects the line of centres of the machine at right angles. The microscope is focussed on the coil and is moved by means of the measuring screw so as to bring the consecutive turns of the coil, one after the other, into a definite position with respect to the cross wires in the eye-piece, the reading of the measuring wheel being taken at each setting. The calibration of the screw is effected by focussing a graduated line standard in the microscope and taking similar readings on the graduations of the bar. The measuring screw and slide are themselves movable on a second slide in a direction parallel to the axis of the microscope, so that by withdrawing them slightly the standard bar can be inserted at any time between the coil and the microscope, and check readings taken on it without disturbing the coil.

The standard employed was an invar metre, the history of which is so well known that the probable error over a length of 12 cm. (the axial length of a coil) is less than 1μ.

For the measurements on each cylinder, readings on successive half centimetre divisions of the invar metre were taken over a length of 14 cm., and readings over a length of 12 cm. (corresponding to the axial length of the coils), were frequently made. Any change in temperature of the screw of the measuring machine could thus be allowed for.

As already explained, the two wires on each cylinder are wound in double screw cut grooves. To measure the mean variation in pitch of the two coils it is better, therefore, to make observations on the spaces between neighbouring wires than on the wires themselves, for in the latter case twice as many observations are required. In practice, observations on the spaces proved the easier way; the white marble showed up well between the wires, and it was not difficult to bisect the white spaces by means of the cross wires of the microscope. In addition, check observations on the wires were always made. For a complete set of measurements 96 readings are required, and three such sets in different axial planes were made on each cylinder.

After correcting the readings, the mean pitch of the coils was calculated within a few parts in 10,000 by dividing the difference of the extreme readings by the number of turns. The approximate pitch thus obtained was 0·065635 inch ($= 0{\cdot}166712_5$ cm.). We multiplied this number by 1, 2, 3, 4, ..., &c., up to 95, and from the resulting products the corresponding pitch readings of a coil were subtracted. If the coil had been perfectly uniform in pitch, the differences would, when plotted, have lain on a perfectly straight line, but, instead, wavy lines result (fig. 13). As an example of the difference readings those for the coils on cylinder No. 1 are given in Table VI.

TABLE VI.—Giving the Difference Readings in Microns for the Coils on Cylinder No. 1.

μ.	μ.	μ.	μ.
16	21	36	32
11	20	31	38
7	24	36	37
12	39	47	38
28	43	51	31
31	25	42	35
23	25	36	33
17	19	25	37
5	15	27	37
8	17	35	44
3	30	45	39
23	45	53	43
27	38	40	42
37	31	35	32
27	25	29	38
22	23	23	33
10	26	28	36
12	31	33	41
15	43	45	47
28	50	44	39
46	52	41	36
42	42	34	44
35	40	30	43
28	31	27	37

From the products resulting when the numbers 1, 4, 7, 10, &c., up to 94 were multiplied by 0·065635, the half centimetre readings for the invar metre were subtracted and the differences when plotted were found to lie on a straight line as they should do if all the corrections have been properly applied. The graphs for the invar metre are shown in fig. 13 and are distinguished as Invar_1, Invar_2, &c., the numbers corresponding to the numbering of the four cylinders. A phosphor-bronze ring is screwed into one end of each cylinder, and these ends are called the "ring ends."

It is clear from fig. 13 that there is a periodic variation in the pitch of all the coils. The distance corresponding to a complete cycle is half an inch, which is equal to the pitch of the leading screw of the lathe on which the cylinders were turned. The cause of the periodicity may be in the leading screw itself, or it may be due to want of parallelism of the surfaces at which the thrust was taken, or it may be due to both of these. Such periodic variations in pitch are present in nearly all screws. In addition to these periodic irregularities, the graphs show others which are taken into consideration when the mutual inductance is calculated of a coil and disc circumference.

The graphs for the coils enable one to place the cylinders in pairs. The lengths at

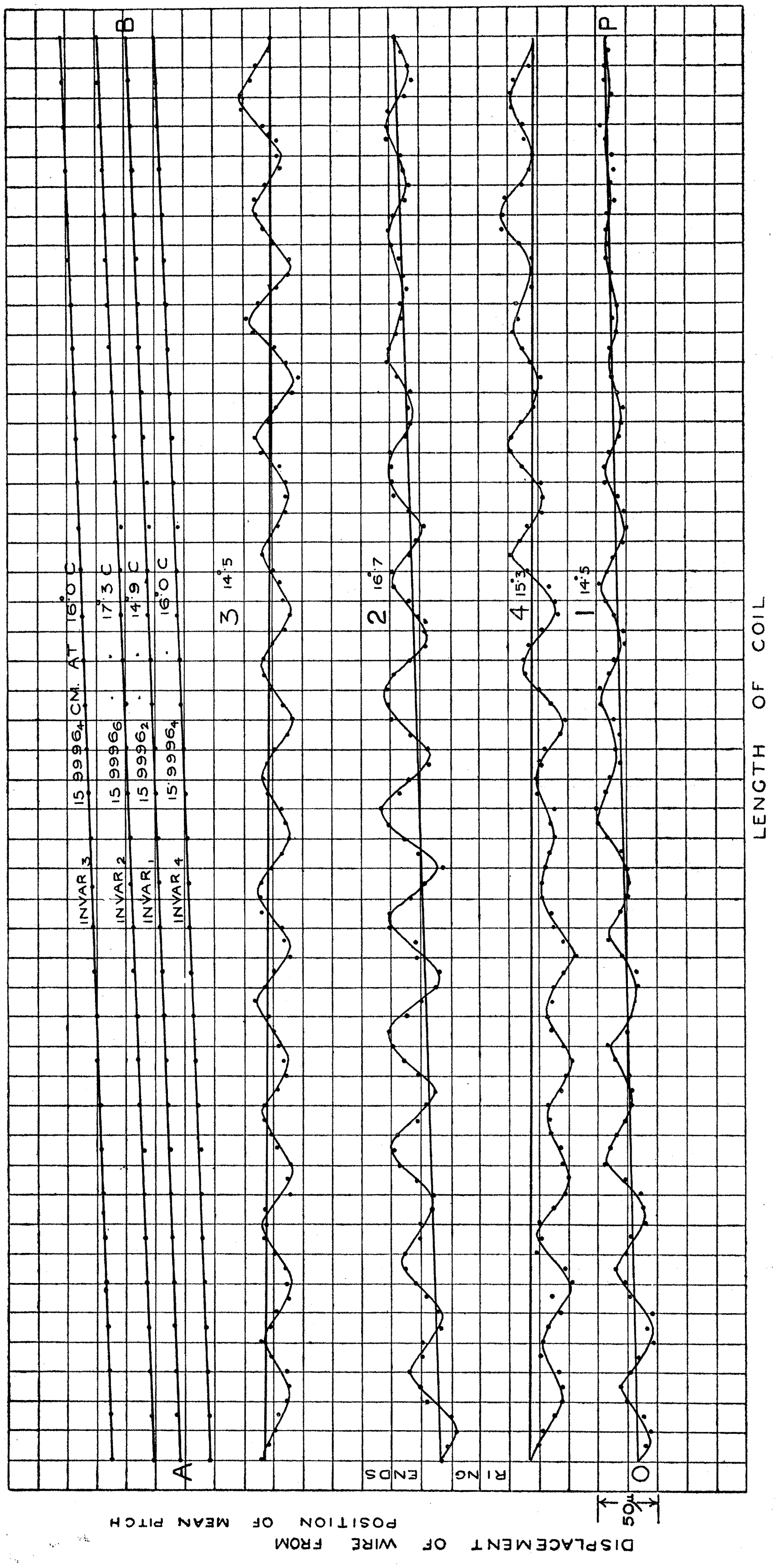

Fig. 13. Showing variations in pitch of coils.

20°·0 C. of the coils on cylinders Nos. 1 and 2 are 16·0018 cm. and $16{\cdot}0013_5$ cm. respectively; those on cylinders Nos. 3 and 4 are 16·0054 cm. and 16·0054 cm. respectively. The amplitude of the periodic variation dies away towards one end for the coils on cylinders Nos. 1 and 2, but for the coils on cylinders Nos. 3 and 4 it does not. The graphs for the variations in diameter of the coils lend support to this division of the cylinders into pairs, and as the same lathe and the same portion of the leading screw were used for the turning of all, the differences appear to be due to a slight difference in the skill and touch of the late Mr. TAYLERSON, who turned cylinders Nos. 1 and 2, and Mr. TRIBE, who was responsible for cylinders Nos. 3 and 4. In all cases the turning is remarkably good.

The mean axial length of the coils on one cylinder is obtained from the graph showing the periodic variation of the pitch and the corresponding graph for the invar metre. Limiting our attention to the coils on cylinder No. 1, it is clear that the extreme mean length of the coils is equal to the length of the invar metre section AB plus the length corresponding to the difference of the ordinates OA and PB. The length of the invar section is $15{\cdot}9996_2$ cm. and the difference of the ordinates corresponds to $+17\mu$. Hence the mean length of the coils on cylinder No. 1 is $15{\cdot}9996_2+17\mu = 16{\cdot}0013_2$ cm. at 14°·5 C. = 16·0018 cm. at 20°·0 C.

Table VII. gives the mean axial length of the coils at 20°·0 C.

TABLE VII.

Coils on cylinder.	Mean axial length at 20°·0 C.
	cm.
1	16·0018
2	16·0014
3	16·0054
4	16·0054

These observations for the mean axial length were made in April, 1913. The results are identical within the limits of the errors of observation (about 3μ) with those obtained in March, 1912. A change in the length of a coil of 20μ produces a change in the mutual inductance of that coil and the two discs of 1 part in 100,000.

SECTION 14.—ERECTING AND ADJUSTING THE INSTRUMENT.

Before assembling the parts of the machine, a concrete block, built up of Keene's cement and Thames ballast, was prepared as a foundation. The block is non-magnetic, is 8 m. long, 80 cm. wide, and 120 cm. deep. Slide rails of gunmetal are bolted in position on this block and to secure greater stability the rails are sunk 5 cm. into the concrete. The rails are in three pairs; one pair support the motor; a second pair the fly-wheel; and the third pair, which are nearly 4 m. long, support the portions with the rotating discs. A period of twelve months was allowed for the concrete block to assume approximately constant dimensions, and at the end of that time the upper surfaces of the rails were scraped to obtain flat surfaces and to ensure these surfaces lying in the same horizontal plane. Two special fitters from Sir W. G. Armstrong, Whitworth & Co. carried out this part of the work, and also superintended the alignment of the parts of the machine. The under surfaces of the castings supporting the fly-wheel, discs, &c., were scraped plane at the Elswick Works and the alignment of the parts was, in consequence, a comparatively easy task. Originally, the couplings were intended to be flexible ones, but after a few runs of the machine rigid couplings were found to be better and such were used. Much of the work connected with the couplings and other fittings to the machine was carried out in the Engineering Department of the Laboratory, and we are greatly indebted to Dr. STANTON, the Superintendent, for much advice on these matters.

At first there was trouble with the bearings. The clearance allowed was very small, and after the machine had run for one or two hours the expansion of the shaft was sufficiently great to cause a collar to come into contact with one of the bearings. Increased clearance was allowed and the difficulty disappeared. From that time (April, 1911) no portion of the machine has given the slightest trouble. The parts are so exactly balanced that the tremor of the concrete base is almost too small to be detected. This is fortunate, as during the observations for an absolute measurement of a resistance, two microscopes are mounted on the concrete bed and used to gauge the distance apart of two coils within a thousandth of a millimetre. These measurements were frequently made when the speed of the machine was 1040 revolutions per minute, but an accuracy within a thousandth of a millimetre was obtained with little trouble. During the progress of our work the machine has been admired by many hundreds of visitors to the Laboratory, and the kindness of Sir ANDREW NOBLE in having the heavy metal work carried out at Elswick is greatly appreciated.

To place the cylinders in position on the cradles, the portions of the shaft supporting the discs were removed from their bearings, the cylinders threaded over the shafts into approximately correct positions and the shafts restored to their proper places. To prevent damage to the cylinders during this operation, a framework of wood was built to support the shafts and cylinders and to enable them to be lifted as one piece. The cradles on which the cylinders are supported are provided with stops and clamping screws to prevent any marked relative movement of a cylinder and the cradle on which it rests.

SECTION 15.—INSULATION TESTS.

When making an absolute measurement of resistance, the greatest difference of potential between any portion of the circuit and the earth was about 130 volts, and the greatest difference of potential between neighbouring turns of wire on any one cylinder was about 16 volts.

The insulation resistance between the circuit and the earth was tested each day on which measurements of resistance were made. For the insulation test, the earth wire

(p. 85, fig. 19) was removed, and the voltage applied to the circuit was the same as that used for the resistance measurements. The insulation resistance was always greater than 1000 megohms.

To measure the insulation resistance between the coils, the latter were usually arranged in two groups, each group consisting of one coil from each cylinder. At first, with an applied voltage of 20 volts, the insulation resistance was about 10,000 ohms. The insulation resistance of coils on cylinders Nos. 2, 3, and 4 proved to be well above 1000 megohms, but that on the coils on cylinder 1 was low. We found these coils to be in close proximity to, or in contact with, a few crystals of pyrites, and after dislodging these, wholly or in part, the insulation resistance increased to above 1000 megohms. No trouble has since occurred. As the coils are of bare copper wire and are covered only with a thin wrapper of silk, we think it necessary to make an insulation test of the coils on each day that resistance measurements are made. The test occupies but a few minutes, and during our work it was regularly made.

The insulation resistance between the earth and the rotating wires attached to the discs was usually tested at 20 volts. The results in all cases were satisfactory. For certain measurements, it is essential that the rotating wires be insulated from each other and as the wires may not occupy exactly the same positions with respect to the disc and shaft when rotating, as when stationary, the tests are preferably made with the machine running. The test when the wires are rotating is of some interest. One terminal of the galvanometer is connected to one brush, and one pole of the battery to the remaining four brushes, the five brushes being in contact with the segments on one of the discs. The other terminal of the galvanometer is connected to the remaining pole of the battery. From fig. 5, given on p. 42, it is clear that when the discs are rotating one terminal of the galvanometer is always connected to one or two of the rotating wires, and the other terminal, through the battery, is connected sometimes with four and sometimes with eight of the wires. The wires are continually changing in position and the deflection of the galvanometer enables the insulation resistance to be calculated. The first set of conducting wires which were used became faulty in insulation resistance because of the action of some insulating tape which was used to bind the wires together. They were replaced by others which were supplied with double layers of silk and which we subsequently shellacked and encased in silk tubes. No trouble has since been experienced, the insulation resistance being well over 200 megohms. The insulation of the remaining part of the circuit, *e.g.*, leads to brushes, standard resistance, &c., was frequently tested, but no fault was found.

SECTION 16.—MEASUREMENT OF THE DIAMETRAL DISTANCE BETWEEN OPPOSITE SEGMENTS.

The distance between opposite segments was measured both when the discs were stationary and when running at speeds varying from 170 to 1110 revolutions per minute.

For the stationary measurements, two micrometer heads were fixed to, but insulated from, two short upright rods of brass secured to a stout bar of the same metal, the distance between the contact faces of the micrometer screws being approximately equal to the distance between opposite segments. This gauge was supported on two uprights secured to the slide rails of the Lorenz apparatus and adjusted in position for the measurement of a diameter. Between the segments and the micrometer screws, vertical wires of phosphor bronze, similar to those used for the brushes, were interposed, and the measured distance was taken as equal to the distance apart of the segments plus twice the diameter of a wire. Contact between a segment and a micrometer head was indicated by the buzzing of a telephone due to the passage of a small alternating current through a circuit including the telephone, the micrometer head, and the segments. Four measurements were made on each pair of segments lying on opposite sides of a diameter and the mean of the 20 measurements was taken as correct. Such a series of measurements was frequently taken and show the wear of the segments, due to their friction with the brushes, to be comparatively slight. As examples, we give the results of some measurements made in December, 1912, and in June, 1913.

TABLE VIII.—Giving the Diametral Distance between Opposite Segments.

Segments.	Disc No. 1.		Disc No. 2.	
	December, 1912.	June, 1913.	December, 1912.	June, 1913.
	cm.	cm.	cm.	cm.
1 and 6	53·581	53·562	53·556	53·546
2 „ 7	·579	·566	·556	·547
3 „ 8	·579	·569	·550	·546
4 „ 9	·579	·573	·556	·549
5 „ 10	·584	·570	·554	·560
Mean	53·580	53·568	53·554	53·550
Mean	53·574 cm.		53·552 cm.	
Mean radial distance of segments . .	= 26·787 cm.		= 26·776 cm.	

The standard of length used was an invar rod with approximately flat ends, and measurements on this rod were made in the same way as those on the segments.

When the discs were rotating, contact with the segments was made by advancing a brush by means of a micrometer screw until a telephone indicated the completion of a circuit as in the previous measurements. The micrometer head made contact with the brush holder through the medium of a small steel hemisphere attached to the holder, and good contact was ensured by a spring attached to the supporting ring and brush, which forced the latter towards the micrometer screw. The brush wires were put under considerable tension and petrol was used as a lubricant. Measurements were made at speeds varying from 170 revolutions per minute to 1110 revolutions per minute, the speed being registered by the directly driven chronograph. The results of three sets of measurements are plotted in fig. 14. The relation between the increase in radius and the square of the number of revolutions per minute is $dr = 3{\cdot}7n^2 \times 10^{-8}$, where dr is the increase in radius in millimetres and n is the number of revolutions per minute. The normal speed of the apparatus when resistance measurements are in progress is about 1050 per minute, and the mean radius is then 0·04 mm. greater than the mean value given in Table VIII. The effective radial distance at 20°·0 C. of the segments is therefore 26·787+0·004 = 26·791 cm. for disc No. 1 and 26·776+0·004 = 26·780 cm. for disc No. 2.

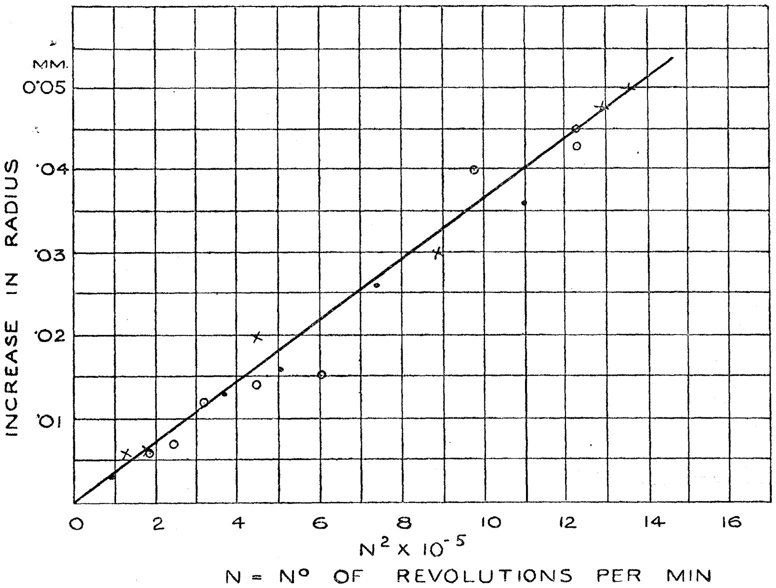

Fig. 14.

SECTION 17.—DETERMINATION OF THE DISTANCE BETWEEN THE CENTRES OF THE COILS.

The distance between two coils on opposite sides of a disc has to be known with considerable precision, as a change of three-thousandths of a millimetre in this distance changes the mutual inductance of the coils and discs by 1 part in 100,000. As it is not possible to make direct measurements of this distance with both rapidity and precision, we followed the plan adopted by GLAZEBROOK* (and by Lord RAYLEIGH in 1882) of reversing the coils, without interchange, and then repeating the resistance

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* 'Phil. Trans.,' vol. 174, p. 254, 1883.

measurements. Reference marks are made on the flanges of the cylinders, and when the coils are parallel the mean distance between two such marks, one on each cylinder, in the two positions of the coils, is exactly equal to the mean distance between the mean planes of the coils. If there are two marks diametrally opposite on each cylinder and two distances are measured for each position of the coils, the mean distance between the marks is the mean distance between the centres of the coils if the latter are *approximately* parallel. In our case a want of parallelism of 0·25 degree (far in excess of that met with in practice) introduces an error of less than 1μ in the determination of the distance between the centres of the coils. In practice we have four marks on each cylinder, these being at approximately equal distances from the mean plane of a coil, and the distance of each mark from such plane is known within 10μ or 20μ. The vertical distances of the marks from the axis of the coil are the same within 0·1 mm., and for the measurements under consideration the distances may be regarded as identical.

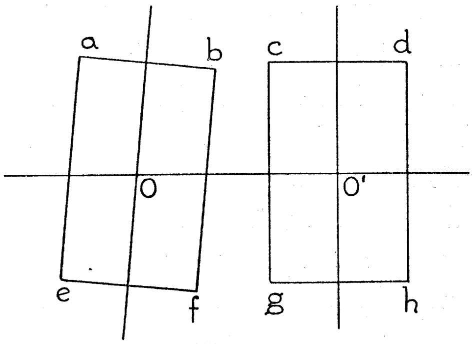

Fig. 15.

Fig. 15 represents diagrammatically two cylinders at a mean distance OO' apart. When the coils are parallel, the distances ac, eg, bd, and fh are approximately equal to OO'. When the coils are reversed but not interchanged, the distances are in general different from what they were before; let the distances be $a'c'$, $e'g'$, &c. With parallel coils each of the mean distances $(ac+a'c')/2$, $(eg+e'g')/2$, &c., is equal to the average distance apart of the mean planes of the coils. If the two coils are not parallel but very nearly so, the average of the distances ac, $a'c'$, eg and $e'g'$ is equal to the average of the remaining four distances and also equal to the average distance apart of the centres of the coils.

In the flanges of each cylinder four brass plugs are screwed and cemented in position. The plugs in each flange are at opposite ends of a diameter, and the four plugs in one cylinder are contained in a common axial plane. Fig. 4 shows two of the plugs ab in cylinder No. 3, and two cd in cylinder No. 4. On the faces of the plugs thin rectangular pieces of platinum are soldered; the surfaces of these are polished, and a scale of half millimetres, cut at right angles by two horizontal lines, half a millimetre apart, is engraved on each piece. The scales were engraved by the late Mr. DONALDSON of the Metrology Department.

For the measurement of the distances, two microscopes are mounted on a special platform supported on a tripod, the base of which can be clamped to the bed of the Lorenz machine by means of the wing nuts w, w', w'', &c., shown in fig. 4, and by means of screws the platform can be given the necessary movements to enable both microscopes to be rapidly focussed without any change in their relative position. When the platform is clamped by the wing nuts ww', the lines on the plugs a and c

are under observation, and when the platform is clamped by means of w' and w'', the lines on plugs b and d can be viewed, and so on for the remaining positions.

The reference standard is a scale of half millimetres, engraved on invar. The scale was standardised by Mr. ATTWELL before making any absolute measurements of resistance, and again after their completion. The results on the divisions employed in our work agree within less than 1μ.

The general procedure in making the measurements was to observe the invar scale, then the eight plug distances, and finally the invar scale again. The first and last of the observations agreed in general within 1μ or 2μ. An absolute measurement of resistance immediately followed and then another set of observations was made on the invar scale and the plugs. The coils were not reversed in position until a large number of resistance measurements had been made.

The distance between coils Nos. 1 and 4 and between Nos. 2 and 3 is required with a much less degree of accuracy, an error of 1 mm. in both of these distances producing a change in the mutual inductance of less than 1·5 parts in 100,000. As the distances between coils Nos. 1 and 2 and Nos. 3 and 4 are known from the previous measurements within a few thousandths of a millimetre, it was only necessary to measure the distance 1 to 4, or 2 to 3. In practice we measured the distance between a plug on cylinder 2 and a plug on cylinder 3, and employed for this purpose a brass bar with half-millimetre divisions engraved at the ends; this enabled the distance to be determined within about 0·1 mm. The distances of the plugs from the mean diametral planes of the coils were known and no further measurements were therefore necessary. Instead of measuring the distance between two coils such as Nos. 2 and 3, we may, as an alternative, measure the distance between the two sets of five brushes, and a number of such measurements were made as checks.

SECTION 18.—CALCULATION OF MUTUAL INDUCTANCE OF THE COILS AND THE CONTACT CIRCLES OF SEGMENTS AND BRUSHES.

The arrangement of the coils and discs in the instrument may be represented diagrammatically in section by fig. 16. The coils are numbered 1, 2, 3, and 4, and the discs are denoted by D_1 and D_2.

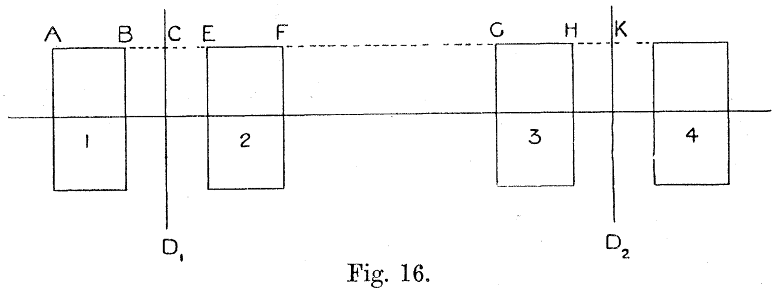

Fig. 16.

The mutual inductance M of any one coil and the contact circles is calculated in two parts. The first part gives the mutual inductance M_1 of the coil and the circumference of the disc nearer to it, and the second part gives the mutual inductance M_2 of the coil and the circumference of the other disc. The difference of these is required, so that $M = M_1 - M_2$.

To find M_1, two mutual inductances were calculated, viz., that between D_1 and a coil of length BC, and that between D_1 and a coil of length AC. The difference of these mutual inductances gives M_1. If the coils on the four cylinders are exactly similar the values of M_1 are identical.

To find M_2 for coils Nos. 1 and 4, the mutual inductances were calculated between D_2 and a coil of length BK and that between D_2 and a coil of length AK. The difference is equal to M_2. To find M_2 for coils Nos. 2 and 3, the mutual inductances required are that between D_2 and a coil of length EK and that between D_2 and a coil of length FK.

In the Lorenz apparatus, the distances of the coils from the discs can be varied. This changes the mutual inductance and the rate of change of M with variation of axial distance must therefore be known.

To find M_1 we used the following formula due to J. VIRIAMU JONES*

$$M = \Theta (A+a)\, ck \left\{ \frac{F-E}{k^2} + \frac{c'^2}{c^2}(F-\Pi) \right\}.$$

In this expression, Θ is the angular length of the helix, A the radius of the helix, a the radius of disc or contact circle, and x the axial length of helix.

$$c^2 = 4Aa/(A+a)^2, \qquad c'^2 = 1-c^2,$$
$$k^2 = 4Aa/(A+a)^2+x^2, \qquad k'^2 = 1-k^2,$$

F, E, and Π, are complete elliptic integrals of the first, second, and third kinds respectively ; F and E are to modulus k, and

$$\Pi = \int_0^{\Pi/2} \frac{d\psi}{(1-c^2\sin^2\psi)(1-k^2\sin^2\psi)^{1/2}}.$$

Putting $c'/k' = \sin\beta$, the quantity $(F-\Pi)$ can be expressed in terms of complete and incomplete integrals of the first and second kinds† ; thus

$$c^{-1}k'^2 \sin\beta\cos\beta\,(F-\Pi) = -\tfrac{1}{2}\pi - F(k)\,F(k',\beta) + E(k)\,F(k',\beta) + F(k)\,E(k',\beta).$$

The various elliptic integrals required were calculated by interpolation from LEGENDRE'S tables, but as a check on possible misprints in the tables a number of the integrals were calculated directly by successive quadric transformation.‡

The dimensions chosen for calculating the values of M_1 are given in Table IX. The

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* J. V. JONES, 'Roy. Soc. Proc.,' vol. 63, p. 198, 1898.

† CAYLEY, 'Elliptic Functions,' § 183.

‡ CAYLEY, Chapter XIII.

dimensions of the coils and discs differ from these values by small amounts $d\mathrm{A}$ for the radius of the coils, da for the radius of the discs, and dx for the length of the coils. Small corrections to the calculated values in Table IX. have therefore to be applied. These corrections are obtained by application of the increment formula

$$\frac{d\mathrm{M}}{\mathrm{M}} = q\,\frac{d\mathrm{A}}{\mathrm{A}} + r\,\frac{da}{a} + s\,\frac{dx}{x},$$

which gives the change in M due to small changes in dimensions, A being the radius of the coil, a that of the disc, and x the axial length of the coil; q, r, and s are coefficients which are given by the expressions

$$q = \frac{\Theta ck}{\mathrm{M}}\,\mathrm{A}\left\{\mathrm{F} + \frac{\mathrm{A}-a}{2a}\,(\mathrm{F}-\Pi)\right\},$$

$$r = \frac{\Theta ck}{\mathrm{M}}\,a\left\{\mathrm{F} - \frac{\mathrm{A}-a}{2\mathrm{A}}\,(\mathrm{F}-\Pi)\right\},$$

$$s = -1 - \frac{\Theta ck}{\mathrm{M}}\,(\mathrm{A}+a)\left\{\left(1-\frac{2}{k^2}\right)\mathrm{F} - \frac{2}{k^2}\,\mathrm{E}\right\}.$$

The sum $q+r+s$ is always equal to unity.

In Table IX. we give in columns 1, 2, and 3 the constants employed for the calculation of nine mutual inductances; in column 4 the values of the mutual inductances; and in columns 5, 6, and 7 the values of q, r, and s, which are required in our work. Table X. gives the difference values M_1.

TABLE IX.—Calculation of Mutual Inductance.

A = radius of coil, a = radius of circle (disc), x = length of coil.

The number of turns per centimetre length of the coil is 12.

A.	a.	x.	M.	q.	r.	s.
cm.	cm.	cm.	cm.			
17·9419	26·7870	23·2650	52702·37	2·1003	−0·5737	−0·5265
17·9419	26·7870	7·2635	23682·39	2·4007	−1·2626	−0·1381
17·9419	26·7870	23·3150	52755·93	2·0998	−0·5724	−0·5274
17·9419	26·7870	7·3135	23822·74	2·3991	−1·2596	−0·1396
17·9419	26·7870	23·2900	52729·18	2·1000	−0·5731	−0·5270
17·9419	26·7870	7·2885	23752·65	2·3999	−1·2611	−0·1388
17·9419	26·7870	$19{\cdot}2851_1$	47883·64	2·1404		
17·9419	26·7870	$11{\cdot}1678_8$	33534·20	2·2880		
17·9419	26·7870	$15{\cdot}2572_5$	41656·11	2·2001		

In Table X. the first difference values are the mutual inductances of a coil 16·0015 cm. long and a circle of radius 26·7870 cm., when the mean diametral plane of the coil is distant from the circle $15{\cdot}2642_5$ cm., $15{\cdot}2892_5$ cm., and $15{\cdot}3142_5$ cm. respectively. In these three positions an increase in the radius of the coil of 10μ increases the mutual inductance by 3·001, 2·995, and 2·988 respectively, and an increase in the radius of the circle of 100μ increases the mutual inductance by −0·125, −0·099, and −0·072 respectively. If we have two such coils, one on each side of the circle, the intensity of the magnetic field at the circumference of the circle will be zero when the mean planes of the coils are about 30·656 cm. apart and the circle is midway between them. The same result is of course brought about by a slight reduction in the diameter of the brush contact circle, an inevitable result of wear and re-turning of the surface of the segments. However, exact realisation of this condition is unimportant since for the positions of the coils dealt with in the calculations, the maximum change in the mutual inductance is only 4 parts in 1,000,000 for a change in the diameter of the circle of one-fifth of a millimetre. Exact centering of the circle between the coils is also unimportant as will now be shown.

TABLE X.—Difference Values of Mutual Inductance.

A = radius of coil	17·9419	17·9419	17·9419	17·9419	17·9419	17·9419
a = radius of circle	26·7870	26·7870	26·7870	26·7870	26·7870	26·7870
x = Axial length of coil . .	23·2650	7·2635	23·2900	7·2885	23·3150	7·3135
M = mutual inductance . .	52702·37	23682·39	52729·18	23752·65	52755·93	23822·74
Difference value = M_1 . . .	29019·98		28976·53		28933·19	
dM for dA = 10μ	+6·169	+3·168	+6·172	+3·177	+6·174	+3·186
Difference value	+3·001		+2·995		+2·988	
dM for dA = 100μ	−11·288	−11·163	−11·281	−11·182	−11·274	−11·202
Difference value	−0·125		−0·099		−0·072	

When the distance of the circle from each of the mean diametral planes of the coils is exactly $15{\cdot}2892_5$ cm. the total mutual inductance of the two coils and the circle is $57953{\cdot}0_7$. If the circle is moved so as to be 0·25 mm. nearer one of the coils the total mutual inductance is increased to $57953{\cdot}1_7$, *i.e.*, about 1 part in 600,000 greater than before. Hence with a brush half a millimetre wide the variation of potential from wire to wire will not exceed 1 part in 600,000. In practice the brushes commonly used consisted of three parallel wires, the extreme width being about 0·5 mm.

Table XI. gives the constants used and the results obtained in one of the calculations dealt with in Table X. and gives also the dimensions of the Lorenz coils and brush contact circles at 20°·0 C. Table XII. gives (1) the differences of dimensions of the coils and standard, and (2) the mutual inductance M_1 of each coil and the contact circle nearer to it.

TABLE XI.

	Standard.	Coils on cylinder No.—			
		1.	2.	3.	4.
A = radius of coil in centimetres .	$17 \cdot 9419$	$17 \cdot 9403_7$	$17 \cdot 9408_0$	$17 \cdot 9427_0$	$17 \cdot 9433_0$
x = length of coil in centimetres .	$16 \cdot 0015$	$16 \cdot 0018$	$16 \cdot 0014$	$16 \cdot 0054$	$16 \cdot 0054$
a = radius of circle in centimetres	$26 \cdot 7870$	$26 \cdot 791$	$26 \cdot 791$	$26 \cdot 780$	$26 \cdot 780$
d = distance of mean diametral plane of coil from the circle in centimetres	$15 \cdot 2892_5$				
n = number of turns	$16 \cdot 0015 \times 12$	192	192	192	192
M_1 = mutual inductance of coil and circle	$28976 \cdot 54$				

We summarise below our knowledge of the changes of M_1 with changes in the dimensions of the coils and disc when the distance of the mean diametral plane of the coil from the brush contact circle is equal to $15 \cdot 2892_5$ cm.

	Change in M_1.
1. Increase in radius of a coil by 10μ	$+2 \cdot 995$
2. Increase in radius of contact circle by 10μ	$-0 \cdot 0099$
3. Increase in length of coil by 10μ, the position of the mean diametral plane remaining constant, and *the number of turns increased so that there may still be* 12 *turns per centimetre*	$+1 \cdot 940$
4. Correcting factor to reduce the mutual inductance to a coil of exactly 192 turns is	16/length of coil in centimetres.

The values of M_1 given in Table XII. hold good only when the mean diametral plane of the coil is $15 \cdot 2892_5$ cm. from the circle. For other slightly different distances we may readily calculate the corrections from the data given in Tables IX. and X. The mutual inductance of the coils on cylinders Nos. 1 and 2 and the brush contact circle No. 1 is 57939·7 when the distance between the mean planes of the coils is 30·5785 cm., and the corresponding values for the coils on cylinders Nos. 3 and 4 is

57955·4, the temperature being 20°·0 C. For an increase in the distance apart of the mean planes of Nos. 1 and 2 by 250μ the mutual inductance diminishes by 43·43, and for a decrease in the distance by the same amount the mutual inductance increases by 43·32. The corresponding values for cylinders Nos. 3 and 4 are practically identical, being 43·48 and 43·37 respectively.

TABLE XII.—Giving the Mutual Inductance M_1 of the Coils and Contact Circles on the Assumption that the Radius is at all Points Equal to the Mean Radius and that the Pitch is Uniform.

Coils on cylinder No.—							
1.		2.		3.		4.	
Differences.	Correction to M_1.	Differences.	Correction to M_1.	Differences.	Correction to M_1.	Differences.	Correction to M_1.
μ.		μ.		μ.		μ.	
$dA = -15\cdot3$	$-4\cdot58$	$dA = -11$	$-3\cdot29$	$dA = +8$	$+2\cdot40$	$dA = +14$	$+4\cdot19$
$dx = +3$	$+0\cdot58$	$dx = -1$	$-0\cdot19$	$dx = +39$	$+7\cdot57$	$dx = +39$	$+7\cdot57$
$da = +40$	$-0\cdot04$	$da = +40$	$-0\cdot04$	$da = -70$	$+0\cdot07$	$da = -70$	$+0\cdot07$
For number of turns n	$-3\cdot26$	—	$-2\cdot54$	—	$-9\cdot78$	—	$-9\cdot78$
Total correction	$-7\cdot30$	—	$-6\cdot06$	—	$+0\cdot26$	—	$+2\cdot05$
$M_1 = 28969\cdot24$		$M_1 = 28970\cdot48$		$M_1 = 28976\cdot80$		$M_1 = 28978\cdot59$	
Sum = 57939·7				Sum = 57955·4			

With change of temperature the mutual inductance varies owing to the expansion of the coils and discs. If the distance apart of the mean diametral planes of the coils is kept constant, the temperature coefficient of M_1 is readily calculated to be $1\cdot06 \times 10^{-5}$ for an increase in temperature of 1° C.

We have previously stated that it is not necessary to know the axial length of a coil with great accuracy. From the data given on page 74, it is clear that if the coils increase in length by 10μ, the change in M_1 for the coils on any one cylinder is $+1\cdot94$. But this supposes the number of turns to increase in the ratio 16·0010/16·0000. Since the number of turns is constant the real change is $1\cdot94 + 28970 - 28970 \times (16\cdot0010/16\cdot0000) = 0\cdot13$. That is, a change of 10μ in the axial length of the coils produces a change in the mutual inductance of the coils and discs of about 5 parts in 1,000,000.

Calculation of M_2, *the Mutual Inductance of the Coils, and the farther Brush Contact Circle.*

The following formula, due to ROSA and GROVER,* gives the mutual inductance of a single layer coil of length x and a co-axial circle of radius A in the plane of one end of the coil:—

$$M = \frac{2\pi^2 a^2 N}{d}\left[1+\frac{3a^2A^2}{8\,d^4}+\frac{5a^4A^4}{64\,d^8}X_2+\frac{35a^6A^6}{512\,d^{12}}X_4+\frac{63a^8A^8}{1024\,d^{16}}X_6+\dots\right],$$

where

$$X_2 = 3-4x^2/A^2,$$
$$X_4 = 5/2-10x^2/A^2+4x^4/A^4,$$
$$X_6 = 35/16-35x^2/2A^2+21x^4/A^4-4x^6/A^6,$$

N = total number of turns in length x,
a = radius of coil,
A = radius of circle,
x = axial length of coil,
$d = \sqrt{x^2+A^2}$.

When the coil is long this formula is very exact and easy to use, and it was a simple matter to calculate M_2 with the precision necessary for our work. The results of 16 calculations are given in Table XIII.

In Table XIV. the values of M_2 are given for coils, the mean diametral planes of which are distant from the brush contact circle by the amounts given in column 4. The results show the variation of M_2 with change (1) in the axial distance of the coils, (2) in the radius of the coils, and (3) in the radius of the contact circle of segments and discs.

The summary on p. 78, Table No. XV., relates to the coils and brush contact circles of the Lorenz apparatus and sufficiently indicates how small corrections were made when the distance between the coils was varied. The contact circle of segments and brushes was practically in the midplane of the coils near to it; the method of ensuring this is described in Section 22.

Correction for Conicality of Coils.

The increment coefficient q (Table IX.) enables the change of mutual inductance to be calculated when the radius of a coil is changed by a small amount, but the change must be a uniform one. When the change is not uniform, the change in mutual inductance for an increase in radius of any part of a coil must be known, and the correction for conicality must be calculated in parts. In the past it has been customary to take the radius of a coil as absolutely uniform from end to end, but such a procedure invariably introduces errors into the calculation.

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* 'Bur. of Standards Bull.,' vol. 8, No. 1, p. 101.

TABLE XIII.—Calculations of Mutual Inductance.

Radius of circle.	Radius of coil of 12 turns per centimetre length.	Length of coil.	Mutual inductance of coil and circle.	Mutual inductance differences = M_2.	Sums of differences.
cm.	cm.	cm.			
26·7870	17·9419	144·1	74981·04	234·26	372·12
26·7870	17·9419	160·1	75215·30		
26·7870	17·9419	174·6	75376·21	137·86	
26·7870	17·9419	190·6	75514·07		
26·7870	17·9419	144·4	74986·12	232·94	370·14
26·7870	17·9419	160·4	75219·06		
26·7870	17·9419	174·9	75379·13	137·20	
26·7870	17·9419	190·9	75516·33		
26·7870	17·9369	144·1	74939·24	234·14	371·93
26·7870	17·9369	160·1	75173·38		
26·7870	17·9369	174·6	75334·19	137·79	
26·7870	17·9369	190·6	75471·98		
26·7370	17·9419	144·1	74985·66	233·42	370·78
26·7370	17·9419	160·1	75219·08		
26·7370	17·9419	174·6	75379·42	137·36	
26·7370	17·9419	190·6	75516·78		

TABLE XIV.—Mutual Inductances M_2.

Radius of circle.	Radius of coil.	Length of coil of 192 turns.	Distance of mean diametral plane of coil from circle.	M_2.
cm.	cm.	cm.	cm.	
26·7870	17·9419	16·0	152·1	234·26
26·7870	17·9419	16·0	152·4	232·94
26·7870	17·9369	16·0	152·1	234·14
26·7370	17·9419	16·0	152·1	233·42
26·7870	17·9419	16·0	182·6	137·86
26·7870	17·9419	16·0	182·9	137·20
26·7870	17·9369	16·0	182·6	137·79
26·7370	17·9419	16·0	182·6	137·36

TABLE XV.—Mutual Inductances M_2.

Distance apart of mean planes of coils.	Distance apart of contact circles.	Radius of contact circle 2.	Radius of coils.	M_2.
cm.	cm.	cm.	cm.	
Giving M_2 of Coils on Cylinders Nos. 1 and 2, with the Contact Circle of Disc 2.				
30·20	167·50	26·7800	17·9406	370·57
30·50	167·35	26·7800	17·9406	371·89
30·50	167·65	26·7800	17·9406	369·91
30·50	167·50	26·7800	17·9406	370·90
30·55	167·50	26·7800	17·9406	370·95
30·60	167·50	26·7800	17·9406	371·01
30·80	167·50	26·7800	17·9406	371·23
Giving M_2 of Coils on Cylinders Nos. 3 and 4, with the Contact Circle of Disc 1.				
30·20	167·50	26·7910	17·9430	370·94
30·50	167·35	26·7910	17·9430	372·27
30·50	167·65	26·7910	17·9430	370·28
30·50	167·50	26·7910	17·9430	371·27
30·55	167·50	26·7910	17·9430	371·32
30·60	167·50	26·7910	17·9430	371·38
30·70	167·50	26·7910	17·9430	371·49
30·80	167·50	26·7910	17·9430	371·60

The change in mutual inductance for an increase in radius of any section of the coil is most readily calculated by finding dM/dA for the brush contact circle and a second circle of the same radius as the coil, the distance apart of the two circles varying over the length of the coil.

TABLE XVI.

d = distance apart of the circles.	Radius of first circle = 26·787 cm. Radius of second circle =			Change in M for change of 10μ in radius of smaller circle.
	cm. 17·9141	cm. 17·9419	cm. 17·9697	
	M =			
cm.				
23·3150	88·8782	89·1121	89·3456	0·0084
19·2851	—	113·5449	113·8412	0·0107
15·2573	—	145·4984	145·9000	0·0145
11·1679	—	187·1127	187·6662	0·0199
7·3135	232·7922	233·5633	234·3348	0·0278

In Table XVI. we give the values of the mutual inductance of two such circles. The radius of one circle is 26·787 cm. and the radius of the other circle varies from 17·9141 cm. to 17·9697 cm. In column 5 of the table we give the change in M for a change in radius of the smaller circle of 10μ.

The values in columns 1 and 5 were plotted and the resulting graph was employed, in conjunction with the values of the diameters used in plotting the conicality curves (fig. 12), to calculate the correction for the conicality of the coils. The method is so obvious that we need only give the results.

Table XVII.—Corrections to M_1 for the Conicality of the Coils.

Coils on cylinder No.	Correction to M_1.	
	Ring end of cylinder near disc.	Ring end of cylinder away from disc.
1	$-0·589$	$+0·942$
2	$-0·571$	$+0·494$
3	$+0·295$	$-0·326$
4	$+0·200$	$-0·153$

The mutual inductance of one coil and a disc circumference is about 30,000 cm., and when cylinder No. 1 is reversed in position a change of 1·5 in M_1, or 5 parts in 100,000, is brought about by the conicality of the coils.

Correction for Variation in Pitch.

The graph of the difference measurements of a coil absolutely uniform in pitch is a straight line such as OP, fig. 17. Such a coil may be called a perfect coil and any short section of it a "perfect section." That there is a difference in the mutual inductance of a "perfect" section such as OAP and a circle, and an actual section such as OBP and the same circle, is easily seen. With the exception of the wires at O and P, every wire in the actual section is farther from the circle than the corresponding wires in the perfect section, and the mutual inductance of the former section will in consequence be the smaller. If the periodic curve is symmetrical with respect to the "perfect" curve, the reverse is true for the next section, but the difference is not so great and hence there is not perfect compensation. As we proceed along a coil with such periodic variations the

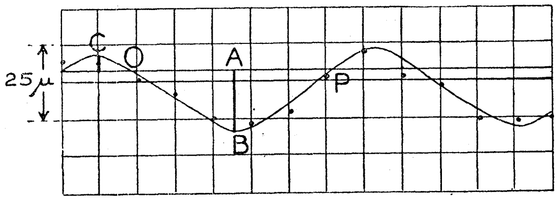

Fig. 17.

individual corrections are alternately plus and minus, and the total effect of these half-period elements depends on the difference of phase at the beginning and end as in the case of diffraction effects in light.

To obtain the corrections we have calculated the mutual inductance of small portions of the coils and the corresponding circles. The mutual inductances calculated are (1) those of the circle and of coils each of which is 0·635 cm. in length, the distance between the circle and the end of the coil nearer to it increasing from 7·3885 cm. to 23·2635 cm. in steps of 0·635 cm.; (2) those of the circle and of coils, each 25μ long, the distance between the coil and circle increasing from 8·0235 cm. to 23·2635 cm. in steps of 1·27 cm. In all cases the number of windings on a coil is at the rate of 12 turns per cm. The results of these calculations were plotted and by means of the curves we have corrected for the variation in pitch.

As an illustration, let us examine the second section of coil No. 3 when the ring end of the cylinder is nearer to the disc. For clearness, this section is reproduced in fig. 17 and is denoted by CB. The end of the section nearer to the disc is 5μ nearer than the end of the corresponding perfect section, and the other end of the section is 20μ farther from the disc than the other end of the corresponding perfect section. The ratio of the lengths of the two sections is therefore 6375/6350 and the ratio of the current densities in the equivalent current sheets is 6350/6375. The M of the perfect section and the disc is 1723·692; if its length is increased by 5μ at the end nearer the disc and by 20μ at its other end, M is increased by the amounts 1·382 and 5·340. (These corrections are obtained from the curve.) The total M is now 1730·414. If next we reduce the number of turns to its original value the M becomes 1730·414 (6350/6375) = 1723·628, and since the section is now identical with the actual section, the correction for variation of pitch and displacement of section is 1723·628 − 1723·692 = −0·064 or −3·7 parts in 100,000.

In this way we have calculated the corrections for all the sections of the coils both when the ring end is nearer to the disc and when farther from it.

The values of the corrections are as follows:—

TABLE XVIII.

Coils on cylinder No.	Correction to M.	
	Ring end of cylinder near disc.	Ring end of cylinder away from disc.
1	+0·502	−0·369
2	+0·987	−0·874
3	−0·734	+0·536
4	−1·535	+0·486

SECTION 19.—EFFECT OF ELECTRIC MOTOR ON THE MUTUAL INDUCTANCE OF THE COILS AND DISCS.

Originally it was our intention to employ a water turbine to drive the Lorenz apparatus, but there were many difficulties in the way of a satisfactory system, and finally we decided on an electric motor. Before coming to this decision we made a number of experiments on the effect of the presence of a large mass of soft iron on the mutual inductance of two coils each of 200 turns of wire and 24 cm. in diameter. The distance between the coils could be changed and the approximate mutual inductance between them was measured by Mr. A. CAMPBELL. The iron employed was a mass built up of laminated sheets tightly pressed together, the dimensions of the mass being 35 × 18 × 8 cm. When the coils were parallel and their mean planes 7 cm. apart, the following values were obtained for the mutual inductance when the iron was placed on the common axis of the coils and at a distance *d* from their mean plane :—

d in centimetres.	M in microhenries.
∞	4942·5
70	4942·6
60	4942·6
39	4945·2
20	4975·0
∞	4942·6

The distance apart of the coils was afterwards increased to 10 cm. and eventually to 20 cm., but in no experiment could the effect of the iron at a distance of 1 m. be detected by the mutual inductance measurements. The stray magnetic field produced by the motor is not a source of trouble and has no influence on our results; the only manner in which the motor can affect the resistance measurements is by its action, as a mass of iron, on the mutual inductance of the coils and discs.

The field magnet of the motor is not large; its length parallel to the shaft is about 30 cm., and its section at right angles to the shaft is about 1000 sq. cm. The distance from the centre of the motor to the centre of the nearest of the four coils is almost exactly 400 cm.

Dr. G. F. C. SEARLE, F.R.S., who has taken great interest in our work and to whom we tender our thanks, has very kindly calculated the effect of a sphere of soft iron on the mutual inductance of a coil and circle, both of which are some distance away. If the radius of the sphere is 20 cm. (corresponding roughly to a mass of iron equivalent to our motor) and the coil and circle are of the same dimensions and the same distance from the sphere as in the Lorenz apparatus, the effect of the sphere on the mutual inductance appears to be about 1 part in 10,000,000 which is, of course, absolutely negligible. This value is in very good agreement with two values determined experimentally.

The intensity of the magnetic force, at a point corresponding to the centre of the motor, which is produced by a current of 2 amperes through the four coils of the Lorenz apparatus, is about 0·0025 C.G.S. units. The iron of the motor becomes magnetised and the result is an increase in the total magnetic flux through the discs. We had to find the ratio of this increase to the flux produced by the current in the four coils. This ratio we have found in two ways. In both of these we magnetised the iron of the motor by winding around it a large solenoid of 16 turns of insulated copper wire. When a current was passed through this coil the magnetising field inside the solenoid was approximately in the same direction as the field due to the four coils, *i.e.*, practically parallel to the shaft. The iron was magnetised and the two discs of the Lorenz apparatus were caused to rotate in the resulting field. Since the mean field in which one disc rotated was not the same as that in which the other disc rotated, a difference of potential was produced between the edges of the two discs. In our experiments we compared the galvanometer deflection produced by this potential difference with that produced by a current in the four coils of the Lorenz apparatus when the mutual inductance of the coils and discs was changed by a known amount. The result found is that when the iron of the motor is placed in a magnetising field of 2·5 C.G.S. units, the total effective magnetic flux through the two discs is 1 part in 10,000 of that due to the current in the four Lorenz coils. With such a small magnetising field as 0·0025 C.G.S. units, it was impossible to measure any effect, but the calculated effect is $\frac{0 \cdot 0001 \times 0 \cdot 0025}{2 \cdot 5}$, *i.e.*, 1 part in 10,000,000.

In the second experiment we wound on a framework of wood a similar coil to that surrounding the motor. Both coils were of 20 cm. radius and had 16 complete turns. When a current of 5 amperes was passed through the coil, the intensity of the axial field was measured by means of a magnetometer at distances of 1, 2, 3, and 4 metres from the mean plane of the coil. Similar measurements were made along the axis of the coil which surrounded the motor, the current through this coil being 5 amperes as before. The results are of interest and are given in Table XIX.

The nearer disc of the Lorenz apparatus is about 400 cm. from the centre of the motor, and we may take the magnetic force due to the motor as uniform over this disc. The area of one face of the disc is 2200 sq. cm. so that the total flux through it due to the iron of the motor was $2200\ (0 \cdot 0010 - 0 \cdot 0003) = 1 \cdot 5$. Had the magnetising force on the iron of the motor been that due to 2 amperes through the four coils of the Lorenz apparatus, the total flux would be reduced to $(1 \cdot 54 \times 0 \cdot 0025)/2 \cdot 5 = 0 \cdot 0015$. The total effective flux through the two discs will be somewhat less than this, but the reduction is not important. When making a resistance measurement, the total effective flux through the two discs is about 24,000, so that the magnitude of the motor effect is $0 \cdot 0015/24000$ = about 6 parts in 100,000,000.

The intensity of the stray field of the motor when running at full load has already

TABLE XIX.—Giving the Intensities of the Axial Magnetic Fields produced by a Current of 2 Amperes in Two Solenoids One of which has the Motor as a Core.

Axial distance from centre of coil.	Intensity of magnetic field.	
	No iron present in coil.	Motor as core of the coil.
cm.	C.G.S. units.	C.G.S. units.
100	0·023	0·09
200	0·0034	0·013
300	0·0009	0·0036
400	0·0003	0·0010

The motor was not running during these measurements.

been stated to cause no trouble. The intensity of the component of this stray field parallel to the shaft of the apparatus and 400 cm. from the centre of the motor, was measured to be 0·0006 C.G.S. units.

The general conclusion is that an electric motor may safely be used for such a purpose as ours.

SECTION 20.—ARRANGEMENT OF CIRCUITS.

(*a*) *Connection to Brushes.*—The brushes are connected to insulated terminals fixed to the phosphor-bronze rings, and double silk-covered copper connecting wires pass from these terminals to a selector switch on the observation table. A diagrammatic representation of the connections is given in fig. 19. The selector switch consists of two conducting arms which enable the observer to complete the circuit through any one of the five pairs of brushes by moving the contacts from stud to stud. When it is desired to place the ten brushes in series and thereby get an induced voltage five times as great as with one pair of brushes, conducting straps are placed in the positions SS SS indicated by the dotted lines and the turning head makes contact with the studs 1′ and 5. Any one of the five pairs of brushes may still be selected and observations may be made to test if the brush contacts are satisfactory. When the ten brushes are placed in series the thermo-electric effects are of course added and particular care has therefore to be taken with the brushes. When trouble is experienced it greatly facilitates the work to select pairs of brushes and so detect the faulty ones.

To place the brushes in two sets of five in parallel, conducting straps connect together each set of five terminals TT, &c., on the selector switch. The position of the turning head is immaterial.

(*b*) *Multiple Commutator and Plug Board* (fig. 19).—This is designed after the manner of the multiple commutator used for the Ayrton-Jones current weigher, and a short description will therefore suffice. A commutator allows of the reversal of the current in both coils on any one cylinder, and the plug board allows of the reversal of the current in any one or more of the helices. Each helix is designated by a number and a letter which are marked on an ebonite bridge at the top of the multiple commutator; the turning heads are also numbered to enable changes to be rapidly made without likelihood of error. The lower commutator reverses the current in all of the coils.

By suitable conducting straps the coils on one cylinder may be placed in parallel with those on another cylinder. In many of our observations the coils on cylinders 1 and 2 were placed in parallel with the coils on cylinders 3 and 4.

(*c*) *The Reversing Switch.*—Preliminary experiments indicated that the make and break of the current through the coils had a considerable inductive action on the galvanometer circuit. The deflection of the galvanometer thus produced amounted at times to 50 cm., on a scale at a distance of 2 m., and prohibited such rapid reversal of the current as we desired to make. Similar inductive action is remarked on by Prof. J. V. JONES in the 'Report of the British Association for 1890.' Had the galvanometer circuit been a stationary one, we might have introduced a compensating system, but this was not possible with the system of rotating conductors we had installed. It was evident that the galvanometer circuit should be broken before making or breaking the current circuit, and as the position of rest of the galvanometer coil on open circuit is, in general, different from that when the circuit is closed, it appeared desirable to have the galvanometer excessively damped when not included in the main circuit. The switch shown in fig. 19 enables this to be done. All the connections are shown in the figure and it is not necessary to describe the switch in detail. On moving from stud 1 to stud 2 the galvanometer circuit is shunted by a negligible resistance and the main galvanometer circuit is broken. On moving to stud 3 the current is broken—on to stud 4 the current is made again but in the reverse direction—and when the movement is continued to stud 5 the galvanometer is again placed in the brush circuit. This switch proved very convenient in practice and greatly facilitated our work.

(*d*) In addition to the commutators already described, a simple commutator was added to reverse the potential leads attached to the standard resistance. Such reversals were made to eliminate electrostatic effects.

(*e*) *Galvanometer.*—This was of the Ayrton-Mather type and was very kindly lent to us for the work by The Cambridge Scientific Instrument Company, Limited. The resistance of the galvanometer is 16·5 ohms and the period of the coil is 5·2 seconds. At a distance of 2000 scale divisions the sensitiveness is 57 divisions per microvolt. The external resistance for aperiodic working is 50 ohms.

When the current through the eight coils of the Lorenz apparatus is 2 amperes and a direct measurement of a resistance of 0·01 ohm is in progress, the difference of potential on the standard resistance is 0·02 volts. In making the measurements the current is reversed, and on reversal there is a change in the rest point of the

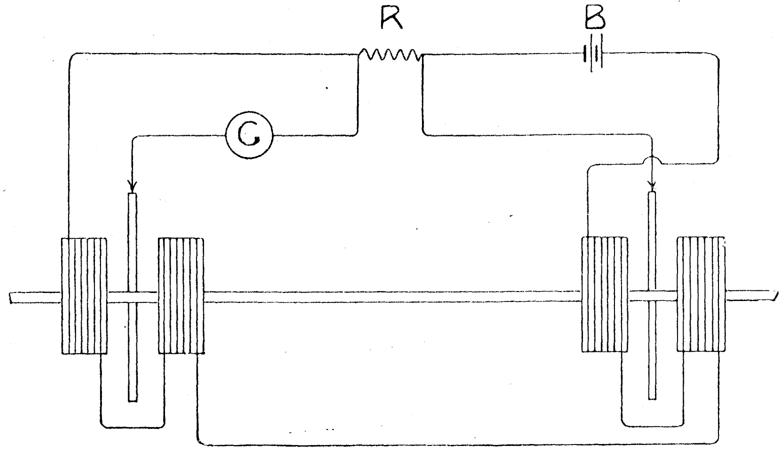

Fig. 18. Simple diagram of circuit.
R, resistance to be measured; B, battery; G, galvanometer.

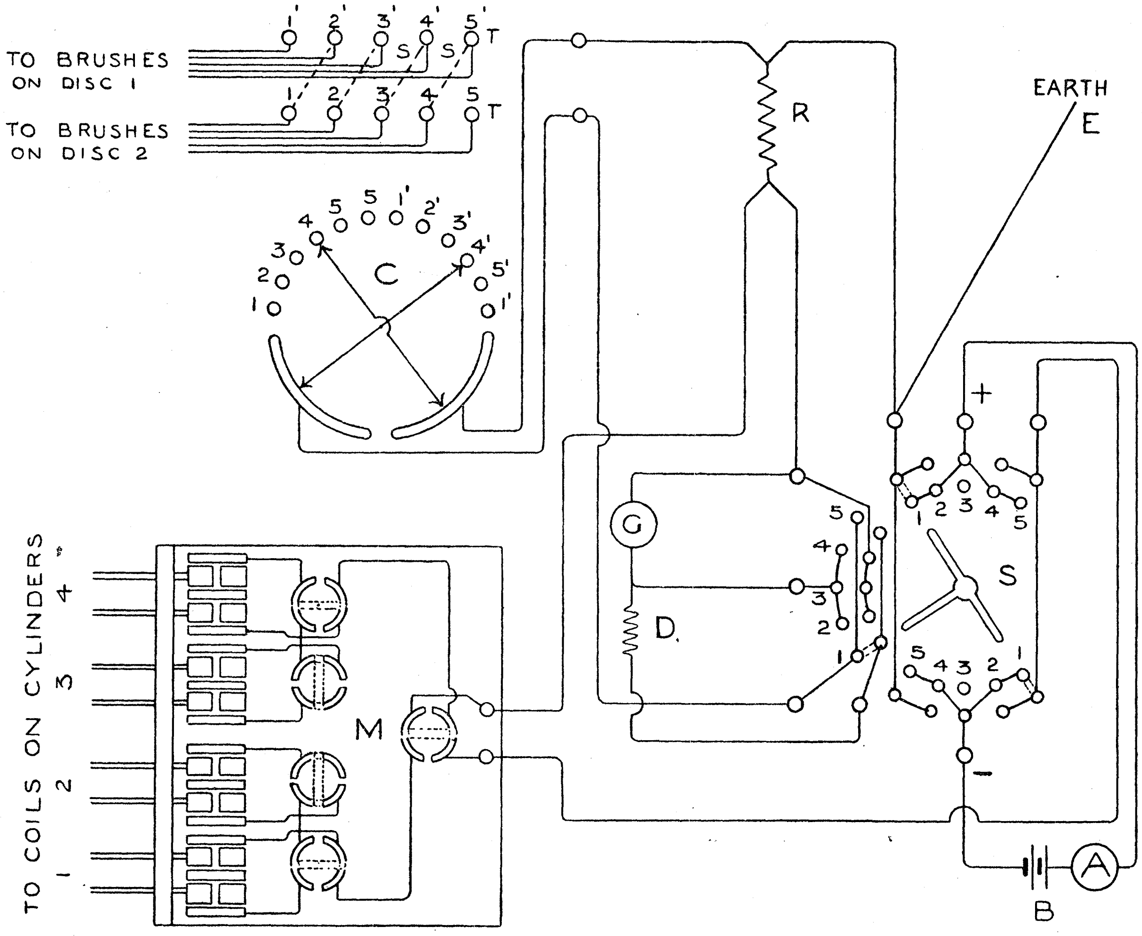

Fig. 19. Detailed diagram of circuit.
A, ammeter; B, battery; C, brush selector switch; D, resistance in galvanometer circuit; E, earthed point on circuit; G, galvanometer; M, multiple commutator and plug board; R, resistance to be measured; S, reversing switch.

galvanometer spot of 57 mm. (at 2 m.) per microvolt. To obtain a precision of 1 part in 100,000 on a single reading it is necessary, therefore, to note the change of deflection when the current is reversed with an error not greater than 11·4 mm. When the motion of the coil is made aperiodic by introducing additional external resistance, the sensitiveness is about one-quarter of the previous value, and the error of the difference reading must not be greater than about 3 mm. For measurements of a resistance of 0·001 ohm the difference of potential on the standard resistance was about 0·004 volt and the change of deflection had to be read with an error not greater than 0·6 mm. For a single measurement of a resistance about fifty different readings were taken and the mean of these is used to calculate the result (see p. 98).

Fig. 18 is a simple diagram of the circuit and requires no explanation. Fig. 19 is a more detailed diagram and shows the connection of the circuit to earth.

SECTION 21.—STANDARD RESISTANCES.

The resistances measured in absolute measure were of nominal values 0·001 ohm, 0·002 ohm, and 0·01 ohm. The first and last of these were standard manganin resistances capable of carrying currents of 30 and 10 amperes respectively without increasing in temperature by an appreciable amount. The currents used in our measurements did not exceed 4 and 2 amperes respectively, and the heating effect due to these currents could not be detected.

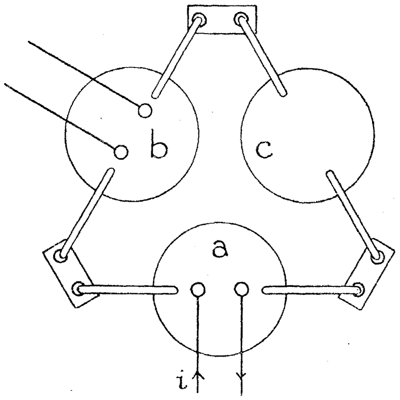

Fig. 20.

To obtain an effect corresponding to that of a resistance of 0·002 ohm, three standard resistances were arranged in a triangular fashion (fig. 20) as first suggested by Lord RAYLEIGH.[*] The resistances are of the well-known type introduced by the Physikalisch-Technische Reichsanstalt; they are of manganin and the mean temperature coefficient in the neighbourhood of 20° C. is about +15 parts in 1,000,000 per degree rise of temperature. The resistance (a) fig. 20 is a coil of 2 ohms resistance, (b) is a 1-ohm standard, and (c) consists of a coil of 1000 ohms shunted by others having values from 100,000 to 500,000 ohms. If the current in the main circuit is i the current through b is $\frac{ai}{a+b+c+\mathrm{L}}$ where L is the summed value of the resistances of the current leads to the three coils. The difference of potential at the extremities of b is $\frac{abi}{a+b+c+\mathrm{L}}$. The quantity $\frac{ab}{a+b+c+\mathrm{L}}$ thus takes the place of R in the formula $\mathrm{R} = \mathrm{M}n$ (p. 35) and is called the effective resistance.[†] The two ohm

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[*] 'Phil. Trans.,' 1883.

[†] A complete treatment on the combinations of a four-terminal resistance with other resistances is given by G. F. C. SEARLE, 'The Electrician,' March 31 to April 21, 1911.

coil is of special construction; it is divided into two equal coils each of which can be compared with the standard coils.

The coils were supported from mercury cups in a bath of well stirred paraffin oil maintained at a constant temperature of 20°·0 C. The stirring was produced by blowing dry air through the oil. A spiral toluene thermostat was used to control the temperature and the heating of the oil was produced by an electric current flowing through a resistance coil supported on a large frame at the bottom of the bath. These arrangements are those in common use in the Electrical Standards Department, and the results obtained are exceedingly satisfactory.

The methods adopted for the accurate comparison of the coils with other wire standards and with mercury standards of resistance are published elsewhere,* and it is not necessary to describe them here. All the resistances were frequently compared with standard manganin coils which are hermetically sealed and the secular changes in which are exceedingly small.

The new mercury standards of resistance of the National Physical Laboratory are nine in number. They have spherical end vessels 4 cm. in diameter and were made in accordance with the specification† of the London Conference on Electrical Units and Standards 1908. The resolution relating to the international ohm is as follows:—

> "The international ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 gr. in mass, of a constant cross-sectional area and of a length of 106·300 cm."

Mercury standards of resistance have also been made at the Physikalisch-Technische Reichsanstalt and the Bureau of Standards at Washington. Recent comparisons show that the units of resistance so derived agree with the unit derived at the National Physical Laboratory within about 2 parts in 100,000. The exact figures are not yet to hand.

SECTION 22.—SETTING OF THE COILS TO BE COAXIAL WITH THE SHAFT AND AT APPROXIMATELY EQUAL DISTANCES FROM THE BRUSH CONTACT CIRCLES.

(1) *Setting of the Axes of the Coils to be Parallel to the Axis of the Shaft.*

Mechanical Method.—At the time of turning the marble cylinders, a phosphor-bronze ring was let into one end of each, and the surface turned at right angles to the axis. When the cylinders were in position on the cradles of the Lorenz apparatus, a radial arm supporting a micrometer head was clamped in a suitable position on the shaft and contact between the micrometer screw and each of the rings in turn was made at three points 120 degrees apart. The making of a contact was

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* 'B.A. Elect. Stands. Committee Report,' 1906.

† 'B.A. Elect. Stands. Committee Report,' 1909.

indicated by a telephone, and readings could be repeated within about 0·01 mm. Each cylinder was adjusted in position until the three readings were identical and the axes of the cylinders were then approximately parallel to the axis of the shaft. The method was sensitive, but irregularities in the thrust-bearing surface of the shaft must introduce errors which are difficult to eliminate.

(2) *Setting of the Coils to be Coaxial with the Shaft.*

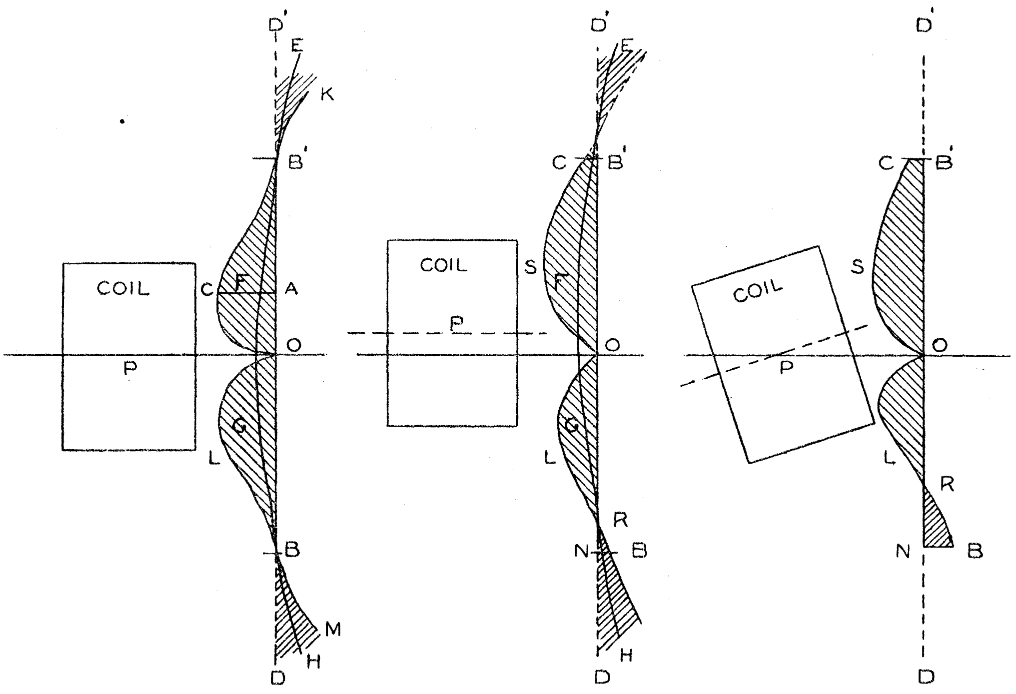
(*a*) *Mechanical Method.*—A direct-reading spring indicator was used to measure the perpendicular distance of the inner surface of each marble cylinder from the surface of the shaft, and each cylinder was adjusted in position until the readings at all points were practically identical. The indicator was sensitive to a difference of about 0·02 mm. and the adjustment of a cylinder occupied only a few minutes. If the inner surface of a cylinder is not coaxial with the coil, then of course an error is introduced.

(*b*) *Electrical Method.*—The calculations we have made give only the mutual inductance of the coils and the brush contact circles, and it is not possible for us to give other than general diagrams indicating the manner in which the intensity of the magnetic field varies along the radius of a circle and other diagrams indicating the difference of potential between the axis and any point on a radius which is produced by the rotation of that radius. Such diagrams will, however, serve to explain the electrical method of setting the coils.

In fig. 21 (*a*), DD′ represents the plane of rotation of one of the discs, and BB′ the position of the brush contacts. The intensity of the magnetic field produced by a current in the coils on both sides of the disc is practically zero at BB′ and changes in sign as we pass radially outwards from the disc. The form of the intensity curve is roughly shown by the curve EFGH. When the disc rotates, the difference of potential between the centre O and any point A on a radius increases with increase of the distance OA to a maximum value at B and B′, but afterwards it decreases. The potential difference is given by $2\pi n \int_0^y \mathrm{B}y\, dy$ where OA $= y$ and B is the magnetic flux at a distance y from O. The potential difference can be represented as an area if the flux be multiplied by y before it is plotted. The area enclosed by the line DD′ and the curve KCLM shows roughly how the potential difference varies; thus the difference of potential between O and A is represented by the area OCA. Areas to the left of DD′ are counted as positive and those to the right are counted as negative. In our apparatus the brush and segment at B are insulated from those at B′ and it is therefore possible to measure the difference of potential between the segments and so determine whether the field produced by the current in the coil is symmetrical with respect to the axis of the shaft. Without the insulation of the segments, *i.e.*, by the use of a disc alone as in the old forms of apparatus, eddy currents in the disc produce disturbing effects.

The effect of displacing the coils parallel to the shaft is diagrammatically shown in fig. 21 (*b*). The area OB′CS is greater than the area OLR−RNB and the difference of potential between O and B′ is greater than that between O and B. The relation between the mean of these voltages and that obtained when the coils and shaft are coaxial will be seen presently.

A similar change in the difference of potential between B and B′ is produced by a rotation of a coil about a vertical axis passing through its centre. Fig. 21 (*c*) shows generally the effect produced and makes it clear that the mean diametral plane of the coil may be set parallel to the brush contact circle by observations of the differences of potential at such points as BB′.



(*a*) (*b*) (*c*)

Fig. 21.

The first coils to be set coaxial with the shaft by these electrical methods were those on cylinders Nos. 1 and 4. Afterwards the mutual inductance of the No. 1 coils and the brush contact circles was made equal to that of the No. 4 coils and the same circles by passing a current in the same direction through both and altering the distance between the No. 1 coils and the disc nearer to them until on reversal of the current in the coils there was no change in the difference of potential between the two contact circles. The object of this procedure was to enable us to measure directly the amount by which a potential difference such as OB′CS, fig. 21 (*b*), exceeded the normal potential difference such as OB′C, fig. 21 (*a*). The circuit formed will be clear from the small diagram at the top of fig. 22. One brush made contact with one (or two) segments on each disc and the circuit was completed through the rotating conducting wires. The displacements were read to 0·001 mm. and the changes in potential were calculated from the changes in the galvanometer deflection which were produced on reversal of the current in the coils.

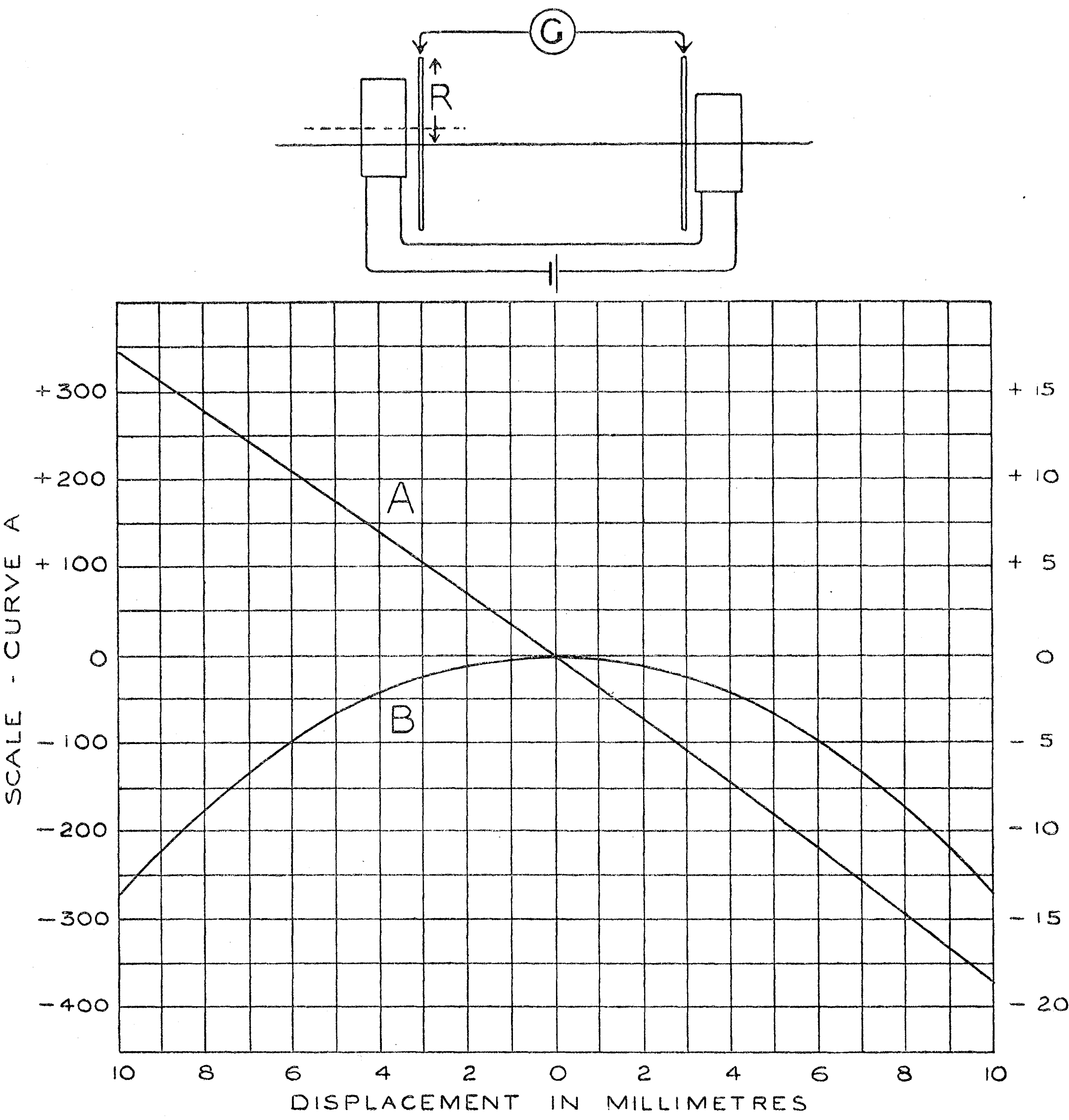


Fig. 22.

A, change (parts in 100,000) in voltage induced by rotation of radius R due to displacement of cylinder in direction perpendicular to its axis; B, change in *mean* voltage induced when five brushes are used, touching points 72 degrees apart on edge of disc.

A displacement of 10 mm. increases the difference of potential between O and B′, fig. 21 (*a*) and (*b*), by about 343 parts in 100,000, and diminishes that between O and B, fig. 21 (*a*) and (*b*) by about 371 parts in 100,000. If one brush only were employed on the segments during measurements of a resistance it would therefore be necessary to ensure coincidence of axes of coils and shaft within about $\frac{3}{100}$ of a millimetre, in order that the error associated with the setting should not exceed 1 part in 100,000.

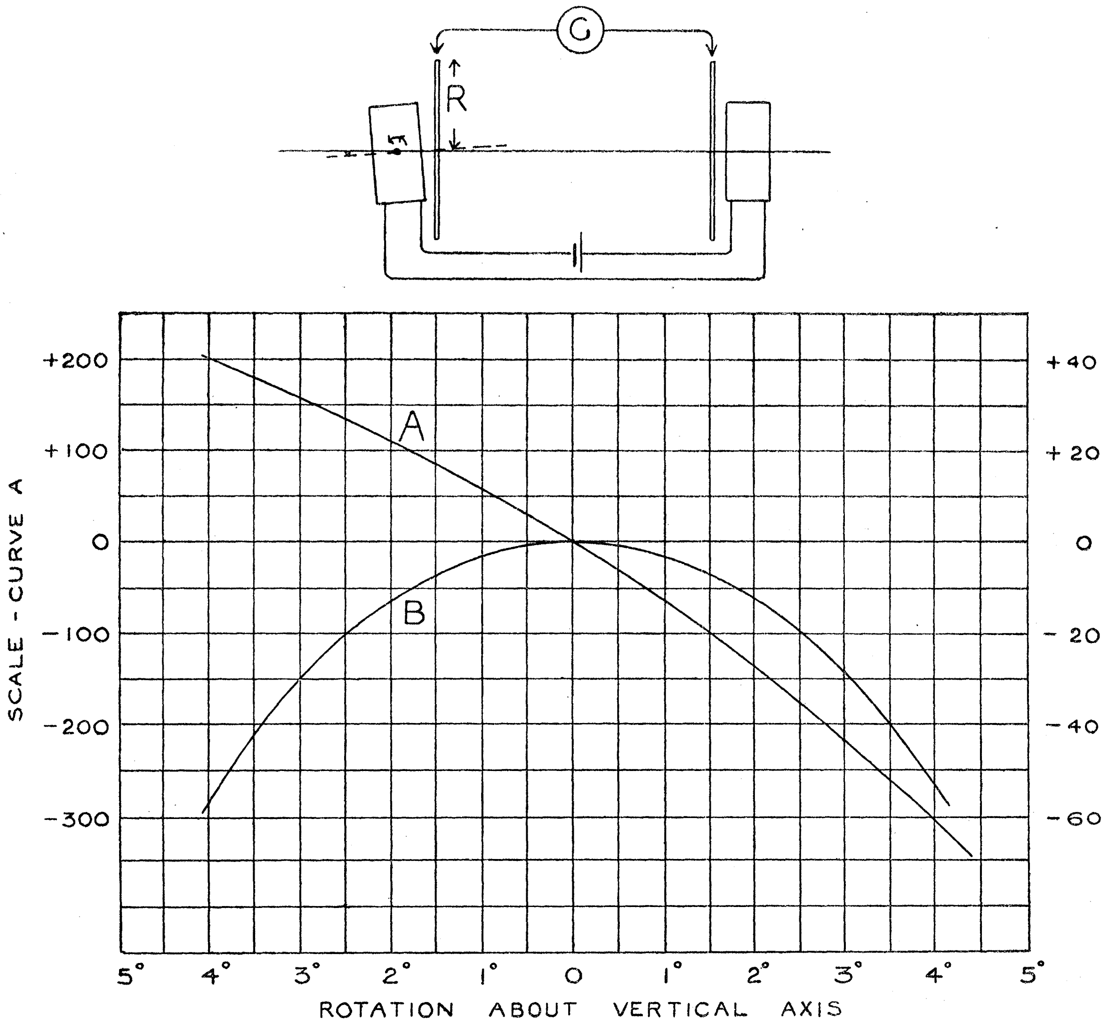


Fig. 23.

A, change (parts in 100,000) in voltage induced by rotation of radius R due to angular displacement of cylinder about vertical axis passing through mean diametral plane; B, change in mean voltage induced when five brushes are used, touching points on edge of disc 72 degrees apart.

In practice, however, five brushes are used on each disc, and when these are used in series with the five brushes on the other disc, a displacement of 10 mm. diminishes the voltage by (371 − 343)/2 = 14 parts only in 100,000. When the brushes on each disc are placed in parallel the mean potential difference measured by the galvanometer is nearly the mean of the separate potential differences and the error of setting is about the same as when the brushes are placed in series. With the brushes in parallel it is better, however, to include a resistance in series with each brush before connecting the brushes together, for then any difference of resistance between the brush contacts is rendered negligibly small. The observed differences of potential due to displacements when the brushes were placed in parallel are plotted in fig. 22, and the equation of the resulting curve is $y = 0{\cdot}132x^2$ where y is the change in voltage in parts in 100,000 produced by a displacement of x mm. from the coaxial position. An identical curve results when the brushes are placed in series. It is clear that the observations enable the coil and shaft to be set coaxial within less than 0·1 mm. and an error of this amount introduces an error in the resistance measurements of about 1 part in 100,000,000.

Similar observations were made when one of the cylinders was rotated about a vertical axis P. The results are plotted in fig. 23. The equation when the brushes are placed in series or in parallel is $y = 3{\cdot}5x^2$ where y is the change in voltage in parts in 100,000 produced by a rotation of x^0. Thus, if $x = 0{\cdot}1^0$ (a large amount) the error introduced is 4 parts in 10,000,000.

(3) *Setting of the Coils on opposite Sides of a Disc to be at the same Mean Distance from the Contact Circle of Segments and Brushes.*

The current through the coils on cylinder No. 1 was caused to circulate in the same direction as the current through the coils on cylinder No. 4, and the resulting flux through disc No. 1 was therefore in the same direction as that through disc No. 2. The galvanometer circuit was completed without the inclusion of the standard resistance R, and reversals of the current through the coils were made in the usual manner to eliminate all effects but that due to a difference of flux through the discs, caused by the current. In general there was a want of balance and in such case the distance of one of the coils was changed until balance was secured or very nearly so. With care equality could be obtained within about 2 parts in 100,000, but in general we were content to ensure equality within about 25 parts in 100,000. This is equivalent to equality, within about 0·07 mm. of the mean distance of the No. 1 coils from No. 1 disc, and that of the No. 4 coils from the No. 2 disc. A similar setting of the No. 2 and No. 3 coils was next made, and afterwards the current through coils Nos. 1 and 3 was made to circulate in the opposite direction to that in coils Nos. 2 and 4. If all the coils are at equal distances from the discs, there is now no deflection of the galvanometer and the positions of coils Nos. 1 and 4 or of Nos. 2 and 3 are changed by equal amounts until a balance is secured. The coils on all of the cylinders are thus placed at the same mean distance, within about 0·1 mm., from the brush contact circles and such equality is sufficient.

SECTION 23.—DETERMINATION OF RESISTANCE IN ABSOLUTE MEASURE.

(1) *Preliminary Tests.*

(A) *Elimination of Error due to the Thermo-electric Effects at the Brush Contacts.*—The thermo-electric effects are eliminated by observing only the change of deflection of the galvanometer with reversal of the current in the coils. If the magnitude of the changes of the thermo-electric effects is considerable, and great accuracy is desired, a large number of reversals are necessary. It becomes important, therefore, to determine the general magnitude of the error introduced by making a comparatively small number of reversals.

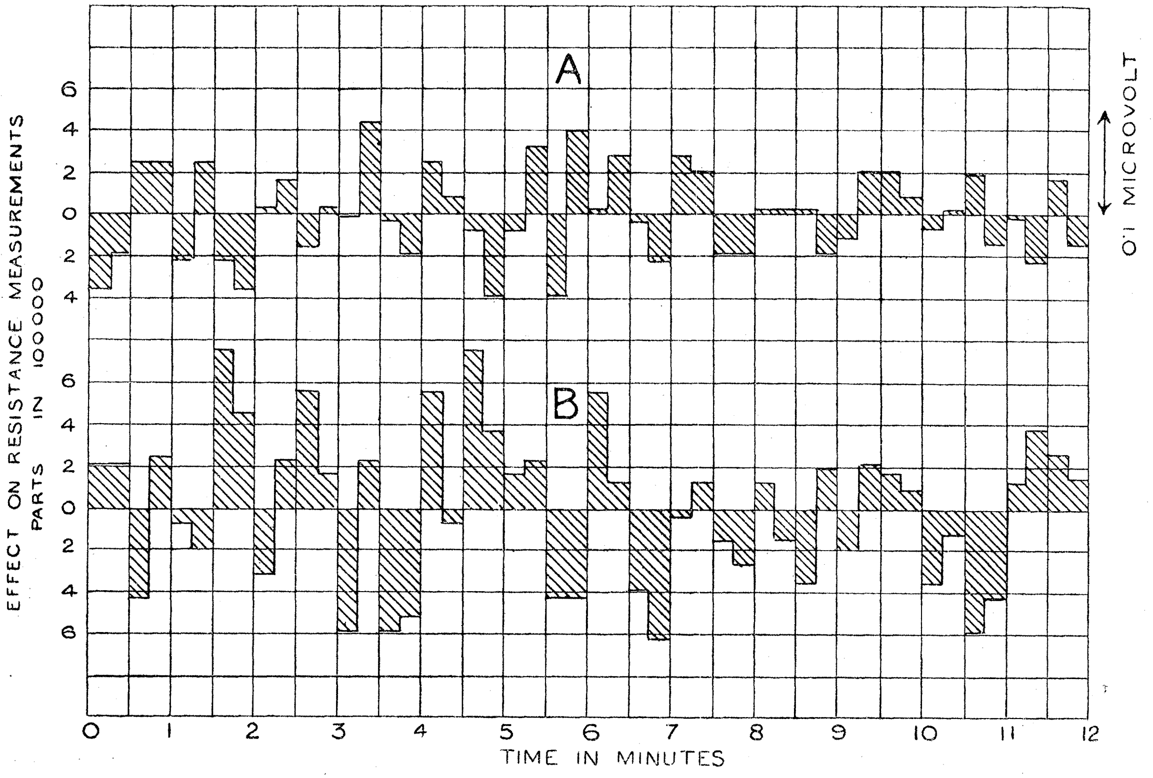


Fig. 24.

A, variations in thermo-electric effects; B, variations in thermo-electric effects *plus* variations in speed.

Our normal procedure in making resistance measurements was to reverse the current in the coils at intervals of 15 seconds, and in practically all cases 48 reversals were made. To determine to what extent this procedure eliminated the thermo-electric effects, we operated the reversing switch with no current flowing in the coils and noted the galvanometer deflection at the end of every 15 seconds.

The differences obtained in one such experiment are plotted against time in graph (A) fig. 24. The mean difference from the mean is ± 0·03 mm., which would introduce an error in a measurement of resistance of something less than 1 part in

1,000,000. The mean difference from the mean irrespective of sign is ±0·8 mm. These differences are typical of those obtained in our experiments and from them we conclude that the thermo-electric effects introduce no error in our final result.

(B) *Elimination of Error due to Electrostatic Effects.*—We have already described the connections of the circuit to earth and the reversals made in order to eliminate any electrostatic effect on the galvanometer, but we thought it desirable to make tests from time to time of the efficiency of our arrangements. In these experiments the coils were removed from the circuit and an equivalent resistance substituted, which resistance was placed in such a position that the current through it produced no magnetic field in the neighbourhood of the discs. The standard resistance remained in the circuit, but the leads connecting it to the galvanometer were connected to one and not both of the potential terminals. Observations of the change of deflection were made as before at intervals of 15 seconds. The mean difference from the mean of such a set of measurements was always negligibly small and indicated that no error due to electrostatic effects could influence our results.

(C) *Constancy of the Speed.*—The records of speed are remarkably good and show the variations to be very small. Each record enables the mean speed, during the measurement of resistance associated with it, to be calculated with great accuracy; but a fluctuation in speed of a few parts in 100,000 lasting for a comparatively short time cannot be detected. The method adopted to control the speed is described in Section 10, and it is evident that the speed must oscillate in value, the magnitude of the oscillations depending on the cause of the variations, the sensitiveness of the bridge, the sluggish movement of the galvanometer, and the ability of the operator. Now the galvanometer in the bridge circuit is sluggish in its movements and unsuited to detect changes in speed lasting for a few seconds only, but the Cambridge galvanometer used in the Lorenz circuit has a short period (5·2 seconds), and the changes in its deflection during a measurement of resistance show that variations in speed lasting for a few seconds only are of common occurrence.

An idea of the magnitude of these sudden changes of speed is afforded by a comparison of the two graphs (A) and (B), fig. 24. Graph (A) shows the variation in the thermo-electric effects, while graph (B) shows the variation in the thermo-electric effects plus the variations in speed. The differences plotted in graph (B) are those taken during a measurement of a resistance. These two graphs are typical of the results generally obtained. In all our experiments the combined variation of thermo-electric effects and changes of speed were of about twice the magnitude of the changes in thermo-electric effects alone, and it appears reasonable to conclude that small oscillations in the speed were frequent. The magnitude of these oscillations appears to be about 5 parts in 100,000. However, such fluctuations had practically no effect on the final results.

(D) *Effect of the Leads.*—It was possible that the current in the leads to and from the coils might produce a magnetic field of sufficient intensity to affect the results.

This was tested by completing the main circuit through the leads only (omitting the coils) and taking a few complete sets of observations. No effect could be measured and we conclude that the current in the concentric leads and remaining portions of the circuit other than the coils have no influence on our results.

(2) *Normal Procedure in making an Absolute Measurement of a Resistance.*—In making a determination of a resistance in absolute measure, we determined (a) the insulation resistance of the various parts of the circuits; (b) the mean distance between the mean diametral planes of the coils; (c) the temperature of the marble cylinders; (d) the want of equality between the product of speed and mutual inductance and the resistance, the value of which was desired; (e) the temperature of the marble cylinder; and (f) repetition of (b).

To measure (b) and (f) we commenced with observations on the invar line standard; we then observed the eight plug distances and concluded with further readings on the invar. These observations lasted about twenty minutes.

The temperatures (c) and (e) of the cylinders were taken by means of thermometers, and the effective temperature was calculated by means of the formula given in Section 13. In general, the difference of temperature $(e)-(c)$ was about 2°·2 C. To determine (d) the commutators and plugs were correctly set, the galvanometer circuit closed, and the reversals of current, &c., made which are indicated in Section 20. At the same time the speed was maintained constant within a few parts in 100,000 and a record of the speed taken. The time occupied was usually from 17 to 20 minutes. In general, the first 12 reversals of current produced changes in the deflection due to a difference $Mn-R$ where R is the value of the standard resistance in absolute measure. The next 8 reversals were made when R was reduced in value by 1 part in 1000 by shunting it with another standard resistance. Then followed 24 more readings with R unshunted; 8 with R shunted, and the final 12 measurements were made with R again normal. The sensitiveness of the arrangement is, of course, directly proportional to the current through the coils, and as this was not constant from day to day we thought it best to determine it on every occasion.

Before proceeding with a measurement, the brushes were wiped with wash-leather and the tension on the wires adjusted. The variations in the thermo-electric effects were then observed and the petrol lubrication attended to. Bad lubrication on disc No. 1 produced a deflection of the galvanometer in one direction, and bad lubrication on disc No. 2 produced a deflection in the reverse direction. With a good supply of petrol on the edges of the discs there was practically no deflection when the brushes were in good condition. When the brushes were somewhat worn—usually after 6 runs—it was not possible to take good observations, and rather than waste time in making indifferent measurements we preferred to wait until new brushes had been inserted. Careful tests of the brushes were regularly made, and it is no doubt due to this fact that we are able to record the result of every completed measurement. At times the thermo-electric and speed variation effects were a little troublesome, but we never considered them sufficiently serious to justify us in discarding the results. It is true that in two experiments we were not able to calculate the resistance; in the first of these, the apparatus for recording the speed was not put in gear and so no record was made; in the second, an interference with the battery connected to the motor produced a change of speed beyond our control and we had to abandon the experiment. In all other cases when a set of measurements was commenced, it was completed and the result is given in this paper.

At a speed of about 17·4 revolutions per second, the product Mn is nearly 2×10^6, and the corresponding resistance is therefore about 0·002 ohm. We have already described the combination of coils which gives such an effective resistance, and we used the apparatus for its measurement on ten occasions.

To measure a resistance of 0·001 ohm, the coils on cylinders Nos. 1 and 2 were placed in parallel with those on cylinders Nos. 3 and 4. The latter coils were found to be equal in resistance to the former within the limits of the errors of the measurements made, and a division of the main current into two parts, equal within about 1 in 3000 could be ensured. In general, equality of the divided currents, within 1 per cent. would have been sufficient. The current through the standard resistance is now twice the mean value of the current through the coils and the resulting equation is $i\mathrm{M}n = 2i\mathrm{R}$. When $\mathrm{M}n = 2 \times 10^6$, R is 0·001 ohm. This arrangement was most convenient. A resistance of 0·001 ohm is more readily compared with 1-ohm standards than a resistance of 0·002 ohm, and as our standard of 0·001 ohm was of thick manganin strip, a current of 4 amperes could be passed through it without an appreciable heating effect.

When the brushes are placed in series, a resistance of 0·01 ohm can be measured, as the equation $5i\mathrm{M}n = i\mathrm{R}$ then holds good. Although this arrangement is very sensitive, a slightly greater pressure is required on each brush and this frequently produced trouble.

A sample series of readings, taken on June 7, 1913, gives a good idea of the measurements involved in a single determination:—

June 7, 1913.

(1) Observations on invar line standard. Temperature = 15°·5 C. Lines 4–616. Length at 15°·5 C. = $30\cdot6016_7$ cm.

Microscope readings (corrected).

	Left.	Right.	Diff. (L − R).	
Taken before readings on plugs	1154·9	946·6	+208·3μ	Mean = +208·3μ.
	1135·0	1028·0	+207·0	
	1133·4	1025·4	+208·0	
Taken after readings on plugs	979·4	770·9	+208·5	
	982·4	774·1	+208·3	
	1035·1	825·2	+209·9	

(2) Observations on plugs.

Microscope readings (corrected).

Plug.	Left.	Plug.	Right.	Diff. (L − R).	Corr.,* *d*.	(L − R) − *d*.
(7)	927·3	(16)	179·2	748·1	− 750	− 1·9μ
(8)	1551·1	(15)	834·1	717·0	− 716	+ 1·0
(4)	1052·0	(11)	1185·2	− 133·2	+ 107	− 26·2
(3)	328·9	(12)	2447·9	− 2119·0	+ 2094	− 25·0
(10)	1042·6	(1)	1232·3	− 189·7	+ 177	− 12·7
(9)	1167·6	(2)	1126·0	41·6	− 55·$_5$	− 13·9
(13)	294·3	(6)	1199·8	− 905·5	+ 887	− 18·5
(14)	1409·9	(5)	757·5	652·4	− 669	− 16·6
					Mean	− 14·2μ

∴ Mean distance between the mean diametral planes of the coils

$$= 30{\cdot}6016_7 \text{ cm.} - 14{\cdot}2\mu - 208{\cdot}3\mu = 30{\cdot}5794_2 \text{ cm.}$$

(3) Temperature of Marble Cylinders.

No.	Before resistance measurements.	After resistance measurements.	Difference.
	° C.	° C.	° C.
1	16·6	18·8	+ 2·2
2	16·55	18·75	+ 2·2
3	16·5	18·8	+ 2·3
4	16·55	18·9	+ 2·35
Mean . . .	16·5$_5$	Mean . . .	+ 2·2$_5$

$$\therefore \text{ Effective temperature} = 16^{\circ}{\cdot}5_5 \text{ C.} + 0{\cdot}42\,(2^{\circ}{\cdot}25 \text{ C.}) = 17^{\circ}{\cdot}5 \text{ C.}$$

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* The correction is equal to the difference of the distances of the plugs from the mean diametral planes of the coils, these distances being deduced from the metrology measurements. Thus we obtained from these measurements:—Distance (d_1) of plug 7 from centre = 9·3892 cm., distance (d_2) of plug 16 from centre = 9·3142 cm. Hence, when the coils are parallel, the distance of plug 16 from plug 7 is greater than the distance between the mean diametral planes of the coils by 9·3892 − 9·3142 = 750μ and a corresponding correction must therefore be applied. For our purpose it is necessary to measure four distances between four pairs of plugs, but, in practice, we measured eight distances between eight pairs of plugs in order to obtain a check. The agreement was always good.

(4) Changes in the Deflection of the Galvanometer produced on Reversal of the Current in the Coils.

Set No. 1.	Sensitiveness (*a*).	Set No. 2.		Sensitiveness (*b*).	Set No. 3.
mm.	mm.	mm.	mm.	mm.	mm.
+4·2	−36·4	+0·2	+6·0	−38·4	+2·2
4·2	40·0	4·4	3·8	38·4	4·2
1·0	38·6	0·2	1·2	36·6	4·0
4·4	38·0	0·6	0·0	40·2	3·6
2·8	41·0	6·0	3·0	41·0	1·4
2·2	37·0	2·8	3·8	36·0	2·6
7·0	40·0	7·0	2·4	37·2	0·0
5·4	36·2	5·0	1·8	36·2	1·0
1·6		4·0	3·8		3·8
4·4		4·4	2·4		5·0
6·0		1·0	1·4		4·4
4·0		1·0	4·2		3·8

Mean of Sets 1, 2, 3 = $+3 \cdot 2_0$ mm.

Mean of Sets (*a*) and (*b*) = −38·2 mm.

∴ Change of deflection for a diminution of the resistance of 100 parts in 100,000

$$= -38 \cdot 2 - 3 \cdot 2 = 41 \cdot 4 \text{ mm.}$$

∴ The resistance is greater than the product M*n* by $\frac{100 \times 3 \cdot 20}{41 \cdot 4} = +7 \cdot 7$ parts in 100,000.

(4) Sets of readings similar to (1) and (2) on the invar standard and the plugs were made after the resistance measurements, but these need not be given here. The mean distance obtained is identical within 1μ with that already given. Such remarkable agreement was in general found between the two sets of measurements that we conclude the expansion of the cylinders does not affect the position of the mean diametral planes of the coils. In such a case, with uniform expansion of the four cylinders, the plug distances keep absolutely constant. Because of this we did not always take a second set of readings. Constancy over seven or eight hours was, however, rarely obtained, for the bed of the machine usually increased in temperature, and its expansion resulted in a change in the distance between the cylinders. Of course, the distance was frequently changed intentionally to alter the value of M.

(5) The temperature of the discs was about 17° C. and the distance between the brush contact circles was $167 \cdot 5_0$ cm. The speed was found from the record to be $17 \cdot 3835_7$ revolutions per second.

(6) Calculation of M*n*. :—

(*a*) The value of M_1 is 115893·7 when the temperature of the coils and discs is 20° C. and the distance between the mean diametral planes of the coils is 30·5785 cm.

(*b*) The value of M_2 is 742·4 for a distance between the brush contact circles of 167·50 cm.

(*c*) When the distance between the diametral planes is increased to $30{\cdot}5794_2$ cm., M_1 is diminished by 3·19; and when the temperature of the coils is diminished to $17^{\circ}{\cdot}5$ C., M_1 is reduced by 3·06.

(*d*) When the temperature of the disc is reduced to 17° C., M_1-M_2 is increased by 0·12.

(*e*) Hence the value of M is

$$\begin{aligned} M = M_1-M_2 &= 115893{\cdot}7-742{\cdot}4-3{\cdot}17-3{\cdot}06+0{\cdot}12 \\ &= 115145{\cdot}2 \text{ cm.} \end{aligned}$$

(*f*) Mn is therefore $115145{\cdot}2 \times 17{\cdot}3835_7 = 200163_5$.

(7) Calculation of R:—

We have

$$iMn = 2iR\,\{1+(7{\cdot}7\times 10^{-5})\}.$$

Hence

$$\begin{aligned} R &= 200163_5/2+7_7 \\ &= 100089_5 \text{ cm./sec.} \\ &= 0{\cdot}00100089_5 \text{ ohm.} \end{aligned}$$

This value of R is on the assumption that the position of the mean diametral planes has been correctly estimated from the metrology measurements. To remove the assumption it is necessary to reverse the coils without interchanging, and this was done after 28 measurements of the 0·001 ohm standard had been made. The effect of reversal is very small and indicates that the assumed mean position of the diametral planes is very nearly correct. The mean result obtained in the two positions is taken as the value of R. When the coils Nos. 1 and 2 are used independently of Nos. 3 and 4, an appreciable difference—about 4 parts in 100,000—is obtained on reversal of the coils. The conclusion is that the estimated position of the mean diametral planes of the coils Nos. 1 and 2 is incorrect by about 0·006 mm. The same is true for the coils Nos. 3 and 4.

In Tables XX. and XXIII. we give the data relating to measurements of a resistance of nominal value 0·001 ohm, and Tables XXI. and XXII. give the results only of the measurements of a resistance of 0·01 ohm and an effective resistance of 0·002 ohm.

The observed values in absolute measure and the values in international ohms (new N.P.L. Mercury Standards of Resistance) are given in Table XXIV.

Probable Errors.

The mean observational error of the results given in Table XX. is about 2 parts in 100,000 for a single observation. This error includes all the errors arising from an inaccurate estimation of the distance apart of the coils, of variations in the speed, of an erroneous estimate of the mean speed, of faulty temperature observations, and the

TABLE XX.—Measurement of 0·001 ohm at 20°·0 C. Part A. (Ring Ends of Cylinders away from Discs.)

D_1 = Mean distance between the mean diametral planes of coils 1 and 2 and of coils 3 and 4.
M = Mutual inductance between the coils and the discs.
n = Number of revolutions per second.
R = Value of resistance in absolute measure, as deduced from the observations.
$(R > Mn)$ = Value deduced from mean change in galvanometer deflection on reversal of current.

The values given for D_1, M, &c., are not quite correct. They are subject to small corrections owing to the exact positions of the mean diametral planes of the coils not being known. These corrections are of exactly the same magnitude in Part B of the table, but they are of opposite sign. The mean value of R (Parts A and B) is the true one.

The distance between the brush-contact circles was practically constant and equal to $167\cdot5_0$ cm.

Date.	D_1.	t_1 = temperature of coils.	t_2 = temperature of discs.	M.	n.	$Mn/2$.	$R > Mn$.	R.	Difference from mean. Parts in 100,000.
	cm.	° C.	° C.			cm./sec.	cm./sec.	cm./sec.	
5.3.13	$30\cdot5794_9$	16·6	17	115146·0	$17\cdot3890_2$	100113_8	-22_4	100091_4	−0·6
5.3.13	$30\cdot5795_5$	18·0	18	115147·4	$17\cdot3888_7$	100114_2	-21_6	92_6	+0·6
5.3.13	$30\cdot5802_0$	21·1	19	115148·9	$17\cdot3892_2$	100117_5	-30_2	87_3	−4·7
11.3.13	$30\cdot5790_3$	16·9	17	115147·9	$17\cdot3812_1$	100070_5	$+19_7$	90_2	−1·8
12.3.13	$30\cdot5782_9$	12·8	13	115145·6	$17\cdot3928_7$	100135_6	-41_8	93_8	+1·8
12.3.13	$30\cdot5951_5$	17·3	15	115092·6	$17\cdot3925_2$	100087_5	$+\ 5_2$	92_7	+0·7
13.3.13	$30\cdot5955_3$	16·5	17	115090·3	$17\cdot3932_2$	100089_5	$+\ 3_3$	92_8	+0·8
14.3.13	$30\cdot5957_9$	17·2	17	115090·2	$17\cdot4024_3$	100142_5	-51_3	91_2	−0·8
17.3.13	$30\cdot5943_1$	15·0	15	115092·7	$17\cdot3928_7$	100089_6	$+\ 3_1$	92_7	+0·7
17.3.13	$30\cdot5924_7$	17·7	17	115102·3	$17\cdot3930_2$	100098_8	$-\ 7_0$	91_8	−0·2
18.3.13	$30\cdot5939_5$	12·2	12	115090·7	$17\cdot3951_7$	100101_1	$-\ 8_8$	92_3	+0·3
18.3.13	$30\cdot5885_9$	15·1	14	115112·7	$17\cdot3951_2$	100120_0	-28_8	91_2	−0·8
18.3.13	$30\cdot5892_6$	16·8	15	115112·4	$17\cdot3945_2$	100116_2	-21_0	95_2	+3·2
19.3.13	$30\cdot5888_0$	13·9	14	115110·5	$17\cdot3939_5$	100111_3	-21_6	89_7	−2·3
27.3.13	$30\cdot5886_9$	14·6	15	115111·7	$17\cdot3896_8$	100087_8	$+\ 3_2$	91_0	−1·0
28.3.13	$30\cdot5888_3$	13·4	14	115109·8	$17\cdot3935_7$	100108_5	-13_9	94_6	+2·6
28.3.13	$30\cdot5886_4$	15·9	15	115113·5	$17\cdot3937_8$	100112_7	-15_5	97_2	+5·2
7.4.13	$30\cdot6180_3$	12·9	13	115008·0	$17\cdot4058_5$	100090_6	$-\ 2_5$	88_1	−3·9
7.4.13	$30\cdot6084_8$	15·8	15	115044·6	$17\cdot4057_3$	100121_8	-31_0	90_8	−1·2
7.4.13	$30\cdot6266_6$	17·7	17	114983·8	$17\cdot4057_8$	100069_1	$+26_1$	95_2	+3·2
7.4.13	$30\cdot6265_4$	19·4	18	114986·3	$17\cdot4060_0$	100072_6	$+17_4$	90_0	−2·0
8.4.13	$30\cdot5291_7$	14·6	15	115318·7	$17\cdot3635_2$	100116_9	-26_3	90_6	−1·4
8.4.13	$30\cdot5289_4$	17·0	17	115322·3	$17\cdot3629_3$	100116_7	-23_6	93_1	+1·1
8.4.13	$30\cdot5298_1$	18·3	18	115320·8	$17\cdot3629_3$	100115_3	-25_9	89_4	−2·6
8.4.13	$30\cdot5303_3$	19·8	18	115320·9	$17\cdot3641_3$	100122_4	-27_2	95_2	+3·2
9.4.13	$30\cdot5298_2$	15·8	16	115317·8	$17\cdot3637_5$	100117_5	-26_6	90_9	−1·1
9.4.13	$30\cdot5308_1$	18·0	17	115317·0	$17\cdot3641_5$	100119_1	-25_4	93_7	+1·7
14.4.13	$30\cdot5294_7$	16·0	16	115319·6	$17\cdot3644_2$	100122_9	-31_2	91_7	−0·3
							Mean . . .	100092_0	±1·8

TABLE XX. (continued).—Part B. (Ring End of Cylinders near Discs.)

Date.	D_1.	t_1 = temperature of coils.	t_2 = temperature of discs.	M.	n.	$Mn/2$.	R > Mn.	R.	Difference from mean. Parts in 100,000.
	cm.	° C.	° C.			cm./sec.	cm./sec.	cm./sec.	
4.6.13	$30\cdot5521_8$	18·6	19	115241·1	$17\cdot3752_5$	100117_1	-22_7	100094_4	+3·2
4.6.13	$30\cdot5996_1$	19·1	19	115077·0	$17\cdot4019_2$	100128_0	-37_2	90_8	−0·4
5.6.13	$30\cdot5998_0$	18·0	18	115075·0	$17\cdot3962_3$	100093_6	-4_3	89_3	−1·9
5.6.13	$30\cdot6012_1$	19·7	19	115072·2	$17\cdot3955_2$	100087_0	$+5_7$	92_7	+1·5
5.6.13	$30\cdot6007_9$	20·1	19	115074·1	$17\cdot3941_2$	100080_6	$+10_9$	91_5	+0·3
6.6.13	$30\cdot5797_4$	16·7	17	115143·0	$17\cdot3867_8$	100098_3	-7_0	91_3	+0·1
6.6.13	$30\cdot5797_4$	18·2	17	115144·9	$17\cdot3862_8$	100097_1	-6_1	91_0	−0·2
6.6.13	$30\cdot5799_9$	18·5	18	115144·3	$17\cdot3861_8$	100096_0	-7_0	89_0	−2·2
7.6.13	$30\cdot5797_3$	15·8	16	115142·0	$17\cdot3839_2$	100081_0	$+8_0$	89_0	−2·2
7.6.13	$30\cdot5794_2$	17·5	17	115145·1	$17\cdot3835_7$	100081_6	$+7_7$	89_3	−1·9
10.6.13	$30\cdot5793_0$	15·3	16	115142·9	$17\cdot3852_2$	100089_2	-2_1	87_1	−4·1
10.6.13	$30\cdot5889_5$	17·2	17	115111·7	$17\cdot3927_7$	100105_6	-13_8	91_8	+0·6
10.6.13	$30\cdot5891_2$	17·5	17	115111·5	$17\cdot3917_2$	100099_4	-7_4	91_7	+0·5
10.6.13	$30\cdot5892_4$	18·9	18	115112·7	$17\cdot3916_8$	100100_2	-8_1	92_1	+0·9
11.6.13	$30\cdot6201_8$	16·3	17	115002·3	$17\cdot4118_3$	100120_0	-26_8	93_2	+2·0
11.6.13	$30\cdot6200_8$	18·2	18	115005·0	$17\cdot4064_2$	100091_3	$+3_0$	94_3	+3·1
11.6.13	$30\cdot6199_9$	18·7	18	115005·9	$17\cdot4062_7$	100091_2	-3_0	89_2	−2·0
11.6.13	$30\cdot6204_1$	18·7	18	115004·4	$17\cdot4068_9$	100093_4	-3_3	90_1	−1·1
11.6.13	$30\cdot6201_3$	20·9	19	115008·1	$17\cdot4071_3$	100098_0	-7_5	90_5	−0·7
13.6.13	$30\cdot6202_0$	17·3	17	115003·5	$17\cdot4036_5$	100074_0	$+17_3$	91_3	+0·1
14.6.13	$30\cdot5445_8$	16·2	16	115264·7	$17\cdot3667_3$	100088_5	$+5_0$	93_5	+2·3
14.6.13	$30\cdot5442_0$	18·6	18	115268·9	$17\cdot3667_3$	100092_2	$+0_8$	93_0	+1·8
14.6.13	$30\cdot5442_9$	20·2	19	115270·5	$17\cdot3660_2$	100089_5	$+3_2$	92_7	+1·5
16.6.13	$30\cdot5449_3$	18·1	18	115265·7	$17\cdot3632_5$	100069_4	$+21_9$	91_3	+0·1
16.6.13	$30\cdot5451_4$	20·2	19	115267·5	$17\cdot3670_1$	100092_6	-1_6	91_0	−0·2
16.6.13	$30\cdot5453_4$	21·9	20	115268·9	$17\cdot3676_6$	100097_6	-5_2	92_4	+1·2
17.6.13	$30\cdot5462_7$	24·0	22	115268·1	$17\cdot3677_2$	100097_2	-3_9	93_3	+2·1
18.6.13	$30\cdot5281_8$	19·1	19	115325·0	$17\cdot3595_3$	100099_4	-10_7	88_7	−2·5
							Mean . . .	100091_2	±1·5

$$\therefore \text{Resistance} = \frac{100092_0 + 100091_2}{2} = 100091_6 \text{ cm./sec. at } 20^\circ\cdot0 \text{ C.}$$

Probable error of observations = 3 parts in 1,000,000.

Resistance in international ohms (N.P.L.) = $0\cdot00100039_3$ at 20°·0 C.

Difference :—Absolute measure − international measure × 10^9 = 52·3 parts in 100,000.

TABLE XXI.—Results of Measurements of 0·01 ohm at 20°·0 C.

Part A (rings away).			Part B (rings near).		
Date.	Measured value of R in cm./sec.	Difference from mean. Parts in 100,000.	Date.	Measured value of R in cm./sec.	Difference from mean. Parts in 100,000.
13.2.13	100092_{20}	+1·8	18.6.13	100093_{20}	+3·5
4.3.13	90_{40}	±0·0	19.6.13	87_{80}	−1·9
28.3.13	88_{30}	−2·1	19.6.13	90_{90}	+1·2
29.3.13	90_{50}	+0·1	19.6.13	91_{50}	+1·8
31.3.13	93_{20}	+2·8	20.6.13	89_{20}	−0·5
31.3.13	88_{60}	−1·8	20.6.13	88_{80}	−0·9
31.3.13	93_{40}	+3·0	21.6.13	91_{50}	+1·8
1.4.13	93_{00}	+2·6	24.6.13	90_{20}	+0·5
1.4.13	91_{90}	+1·5	25.6.13	86_{70}	−3·0
3.4.13	88_{50}	−1·9	26.6.13	87_{50}	−2·4
4.4.13	87_{80}	−2·6	26.6.13	90_{20}	+0·5
5.4.13	87_{50}	−2·9	27.6.13	89_{20}	−0·5
Mean . . .	100090_{40}	±1·9	Mean . . .	100089_{70}	±1·5

$$\therefore \text{Resistance} = \frac{100090_{40} + 100089_{70}}{2} = 100090_{10} \text{ cm./sec. at } 20^\circ\cdot 0 \text{ C.}$$

Probable error of observations = 5 parts in 1,000,000.

Resistance in international ohms (N.P.L.) = $0\cdot0100038_3$ at 20°·0 C.

Difference :—Absolute measure − international measure × 10^9 = 51·8 parts in 100,000.

TABLE XXII.—Results of Measurements at 20°·0 C. of an Effective Resistance of 0·002 ohm.

Part A (rings away).					Part B (rings near).				
Date.	R in cm./sec.	R in international ohms.	Difference. Abs. − int. (10^9) parts in 100,000.	Difference from mean.	Date.	R in cm./sec.	R in international ohms.	Difference. Abs. − int. (10^9) parts in 100,000.	Difference from mean.
6.3.13	200479_7	200368_2	55·8	+1·8	28.1.13	200469_0	200365_0	52·0	±0·0
6.3.13	472_3	368_2	52·1	−1·9	28.1.13	199457_9	199354_0	52·0	±0·0
6.3.13	472_4	368_2	52·1	−1·9	4.2.13	200465_4	200365_0	50·2	−1·8
7.3.13	478_6	368_2	55·2	+1·2	5.2.13	468_6	365_0	51·8	−0·2
7.3.13	478_1	368_2	55·0	+1·0	6.2.13	473_7	365_0	54·3	+2·3
		Mean . .	54·0	±1·6			Mean . .	52·0	±0·9

Probable error of observations = 6 parts in a million.

Difference :—Absolute measure − International measure × 10^9 = (54·0 + 52·0)/2 = 53·0 parts in 100,000.

Table XXIII.—Results of Measurements at 20°·0 C. of a Resistance of 0·001 ohm (*a*) when the Coils on Cylinders 1 and 2 only were employed; (*b*) when the Coils on Cylinders 3 and 4 only were employed.

(*a*) Coils on cylinders 1 and 2 used.				(*b*) Coils on cylinders 3 and 4 used.			
Date.	Ring position.	R in cm./sec.	Difference from mean. Parts in 100,000.	Date.	Ring position.	R in cm./sec.	Difference from mean. Parts in 100,000.
15.4.13	Away	100089_7	1·1	15.4.13	Away	100093_7	1·1
15.4.13	„	87_4	1·2	15.4.13	„	95_9	1·1
	Mean . . .	**100088_6**			Mean . . .	**100094_8**	
17.6.13	Near	100092_1	0·1	17.6.13	Near	100089_1	0·4
18.6.13	„	92_4	0·2	18.6.13	„	88_4	0·3
	Mean . . .	**100092_2**			Mean . . .	**100088_7**	
Grand mean = 100090_4 cm./sec.				Grand mean = 100091_8 cm./sec.			
Resistance in international ohms = $0·0100039_3$. Difference :—Absolute value – international value × 10^9 = 51·1 parts in 100,000.				Resistance in international ohms = $0·0100039_3$. Difference :—Absolute value – international value × 10^9 = 52·5 parts in 100,000.			

Table XXIV.

No. of observations in each position of coils.		Resistance.			Difference. Parts in 100,000 (abs. × 10^{-9}) – (int.).
		Standard.	Absolute measure.	In international ohms.	
		ohm.	cm./sec.		
28		0·001	100091_6	$0·00100039_3$	$52·_3$
12		0·01	100090_{10}	$0·0100038_3$	$51·_8$
5		0·002	See Table XXII.		$53·_0$
2	Coils 1 and 2 used	0·001	100090_4	$0·00100039_3$	$51·_1$
2	Coils 3 and 4 used	0·001	100091_8	$0·00100039_3$	$52·_5$

The agreement is most satisfactory. Weighted mean . . $52·_0$

error arising from the observation of the galvanometer deflections. The probable observational error of the mean value in absolute measure of the 0·001 ohm resistance standard is about 3 parts in 1,000,000.

The probable error of the diametral dimensions of the coils has been estimated in Section 13 to be not greater than 1μ plus the errors of the gauges employed. Inspection of the results given in Table III. shows that the mean value taken as correct differs from the results obtained in February and March, 1912, by about $1·4\mu$, and from those obtained in April, 1913, by about $0·7\mu$. The gauges employed are believed to be accurate within $0·5\mu$, so that the maximum probable error of the diametral dimensions is about $\pm 1·5\mu$ which corresponds to a probable error in the mutual inductance of about 8 parts in 1,000,000. This assumes that the error is of the same sign for all the coils.

The probable error of the mean axial lengths of the coils has been estimated in Section 13 to be about 3μ, and a change of this amount in all the coils and in the same direction produces a change in the mutual inductance of 2 parts in 1,000,000.

Any error due to lack of knowledge of the position of the mean diametral planes of the coils is practically eliminated by the reversals. An analysis of the plug measurements shows that the error cannot be so great as 2 parts in 1,000,000, and we may therefore dismiss it from consideration.

The growth in the dimensions of the coils due to the passage of the current is not really large. The mutual inductance of the coils and brush contact circles is nearly 3 parts in 100,000 greater after the current has been left on for 20 minutes, but the mean mutual inductance is only about 1·5 parts in 100,000 greater than that at the start. The error of estimation of the increase must have an exceedingly small effect on our final results and need not be considered.

The mean diametral distance between opposite segments on the discs diminished by 0·08 mm. during our experiments. Such a change would be a serious one in the old form of Lorenz apparatus, but in our instrument the change in the mutual inductance due to such a reduction in the mean diameter is less than 4 parts in 1,000,000.

Evidence in favour of a small probable error is afforded by the satisfactory agreement between the results obtained when coils Nos. 1 and 2, and 3 and 4, were used independently. The resistance of the 0·001 ohm standard was found to be 100090_4 cm./sec. when coils Nos. 1 and 2 only were used, and 100091_8 cm./sec. when coils Nos. 3 and 4 were used. The difference is 14 parts in 1,000,000 and part of this is possibly due to the errors of observations, for only four observations were made in each case.

The electrical method of setting the coils in position has been shown to be subject to errors less than 1 part in 1,000,000; the magnetic susceptibility of the parts of the apparatus, excluding the motor, is too small to be measured with the apparatus at our disposal; and the effect of the motor on the mutual inductance of the coils and discs has been proved to be negligibly small. The errors of speed cannot be all of one sign and must be quite negligible apart from a constant error in the clock rate. But this latter is clearly impossible in the case of a standard clock the error of which is taken daily. The daily rate was small and time comparisons were made with another standard clock the daily rate and error of which were also known. At any hour of the day the difference between the clocks agreed, within the possible error of the observations, with that calculated from the errors and rates of the clocks. This agreement is evidence that both clocks were going uniformly or that there was a similar want of uniformity in the going of both. Such similarity is very improbable. As an additional precaution the resistance observations were made at times ranging from 9 a.m. to 6 p.m. but no systematic differences were observed.

The possibility of error due to our coils being of wire of finite section has not been overlooked. The formula developed by J. VIRIAMU JONES gives the mutual inductance when the coils can be treated as infinitely fine helical filaments, and small corrections may be necessary. The case of the coils of the Ayrton-Jones current balance was examined by Dr. G. F. C. SEARLE, F.R.S.,* who showed the correction to be negligible in that instrument. In the present instance, no special treatment is necessary as the mutual inductance calculations made during the work are sufficient to show that no correction need be made. To illustrate this, consider the mutual inductance M of a helical filament coinciding with the axis of the wire and the nearer brush contact circle. Next consider two helical filaments of the same diametral dimensions as the previous one, but let one helix be nearer to, and the other farther from, the circle by 0·25 mm. Table X. shows that the mean mutual inductance of these two helical filaments and the circle is greater than that of the central helical filament by 2 parts in 1,000,000. The wire with which the coils are wound is about 0·557 mm. in diameter, so that strictly we ought to consider filaments 0·28 mm. away from the central one, but in view of the result obtained we think it unnecessary to calculate the small and certainly negligible difference. Next consider two circles coaxial with the disc and at equal distances from it, and let the diameter of one circle be greater than that of the other by 0·556 mm., the mean diameter being 35·88 cm. At distances of 7·3135 cm. and 23·3150 cm. the mean mutual inductance of these two circles and the circumference of the disc does not differ by more than 2 parts in 1,000,000 from the mutual inductance of the disc and circle of radius equal to the mean of the two previous ones (see Table XVI.). The coil of wire may therefore be treated as a helical filament.

The determination of a resistance in absolute measure is therefore subject to a number of small errors, the greatest of which is associated with the determination of the mean radius of the coils. This error is probably not greater than 1 part in 100,000, and if the remaining errors were all of the same sign it is unlikely that their sum would exceed another part in 100,000.

We believe, therefore, that the absolute measurements of resistance which we have made are correct within 2 parts in 100,000.

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* 'Phil. Trans.,' A, 207, pp. 541–544.

The resistances which we have employed have been compared with nine new mercury standards of resistance constructed in accordance with the Specifications of the London Conference on Electrical Units and Standards (1908). These mercury standards of resistance practically realise the international ohm then defined, and the agreement between the nine standards is very good.* There are, however, certain sources of error in the construction of such standards which must always be of the same sign, and the probable error associated with the practical realisation of the international ohm has been estimated to be not less than 2 or 3 parts in 100,000.

The mean of the results given in Tables XX., XXI., XXII., and XXIII. leads to the conclusion :—

A resistance of 1 international ohm is equal to 1·00052 ± 0·00004 ohm (10^9 cm./sec.), the probable error ± 0·00004 being approximately the sum of those involved in the realisation of the ohm and the international ohm.

The international ohm, as defined by the London Conference on Electrical Units and Standards (1908), is the resistance at 0° C. of a column of mercury, 14·4521 gr. in mass of a constant cross-sectional area and of a length of 106·300 cm. As stated in the Introduction, the cross-section of such a column is equal to 1 sq. mm. or nearly so.

Since the international ohm is equal to 1·00052 ± 0·00004 ohms, the mass of the column of mercury of the same cross-sectional area as the international ohm and having a resistance of 1 ohm will be $\frac{14{\cdot}4521}{1{\cdot}00052 \pm 0{\cdot}00004} = 14{\cdot}4446 \pm 0{\cdot}0006$ gr.,† while the length of the column will be $\frac{106{\cdot}300}{1{\cdot}00052 \pm 0{\cdot}00004} = 106{\cdot}245 \pm 0{\cdot}00004$ cm.†

We may sum up our results by stating that :—

The ohm 10^9 cm./sec. is represented by the resistance at 0° C. of a column of mercury 14·4446 ± 0·0006 gr. in mass, of a constant cross-sectional area (the same as for the international ohm) and having a length of 106·245 ± 0·004 cm.

The Historical Introduction shows a number of determinations, notably those of RAYLEIGH (corrected values marked (S) Table I.), GLAZEBROOK (corrected value (S) Table I.), WIEDEMANN, DORN, and HIMSTEDT, in close agreement with that now obtained. These results are as follows :—

1882, RAYLEIGH	106·26 cm.
1882, GLAZEBROOK	106·25 ,,
1883, RAYLEIGH	106·24 ,,
1885, WIEDEMANN	106·265 ,,
1889, DORN	106·243 ,,
1892, HIMSTEDT	106·259 ,,

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* 'Report of the National Physical Laboratory for 1912.'

† The probable errors in these two values are so related that an error in either value is necessarily associated with an equivalent proportionate error in the other.

On the whole, the remaining results in Table I. give the length of the column of mercury representing the ohm as greater than 106·25 cm., and while a discussion of the methods and apparatus might do much to explain some of the differences, much greater detail of the apparatus than is readily accessible is necessary for this to be done.

The ohm and the international ohm differ by about 5 parts in 10,000, and since the ampere (10^{-1} C.G.S. unit) has been realised with considerable accuracy, we may express the electromotive force of the Weston normal cell in absolute measure. The value found in 1908 by Ayrton, Mather and Smith at the National Physical Laboratory for the E.M.F. of the Weston normal cell in terms of the ampere (10^{-1} C.G.S. unit) and the international ohm is 1·01830 volts at 17° C., with a probable error of 2 parts in 100,000. The deduced value at 20° C. is 1·01818 volts. Since that time measurements have been made by Janet, Laporte, and Jouast[*] at the Laboratoire Central d'Électricité; by Prof. Guillet[†] and Prof. Pellat,[†] by Haga,[‡] and by Rosa and Dorsey[§] at the Bureau of Standards. The results are :—

Ayrton, Mather, and Smith	1·01818
Janet, Laporte, and Jouast	1·01836
Guillet	1·01812
Pellat	1·01831
Haga	1·01825
Rosa and Dorsey	1·01822
Mean	1·01824 at 20° C.

These results include the errors of the resistance standards employed as well as the errors of the determination of the current in absolute measure. In addition small differences existed between the Weston cells. Considering all these circumstances the agreement is very remarkable and testifies to the great care taken in the measurements. There appears to be little doubt that the value 1·01824 at 20° C. is correct, within 1 part in 10,000. We conclude that the value of the cell in volts (10^8 C.G.S. units) is 1·0188 ± ·0001 at 20° C. This value may serve for the present for those engaged in absolute measurements.

Section 24.—Conclusions.

From the measurements recorded in the previous sections, we conclude that the instrument we have described may be used for absolute measurements of resistance

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[*] 'Bull. de la Soc. Internat. des Electriciens' (2), vol. 8, p. 459, 1908, and 'Comptes Rendus,' 153, p. 718, 1911.

[†] 'Bull. de la Soc. Internat. des Electriciens,' 1908.

[‡] 'Konink. Akad. Wetensch. Amsterdam Proc.,' p. 587, 1910.

[§] 'Bull. Bureau of Standards,' vol. 8, p. 269, 1912.

with a precision satisfying all present demands whether purely scientific or technical. We believe the instrument fully realises the desires of those who were responsible for such measurements being made, and the results justify Lord RAYLEIGH's belief that the ohm, as defined in absolute measure, can be realised with a precision comparable with that of the international ohm. The instrument can be used at any time; the dimensions of all its parts can be checked when desired, the probable error associated with a single measurement is small, and the observations do not unduly tax the experimenter. We have formed an estimate, based on the measurements already made, of the time necessary to devote to a complete re-determination supposing the cylinders to be stripped of the coils. The re-winding of the coils would occupy three days, the metrology measurements would extend over six days, and sufficient resistance observations could be made in eight more days. Absolute measurements of resistance may therefore be placed in the front rank of precision measurements.

In conclusion, we desire to express our sincere thanks to the Drapers Company of London for providing a large sum of money towards the cost of the instrument, and to Sir ANDREW NOBLE, F.R.S., for his generous help with the heavy metal work.

Our most hearty thanks are hereby tendered to our Director, Dr. R. T. GLAZEBROOK, C.B., F.R.S., who has not only given his very valuable help and advice throughout the work, but has fully appreciated the many difficulties which have arisen and which required much time and patience to remove.

Our best thanks are also due to Lord RAYLEIGH for his keen interest in the investigation, to Dr. STANTON for superintending the turning of the marble cylinders, and to many of the staff of the National Physical Laboratory, particularly Mr. A. CAMPBELL and Mr. DYE, for suggestive aid throughout the investigation.

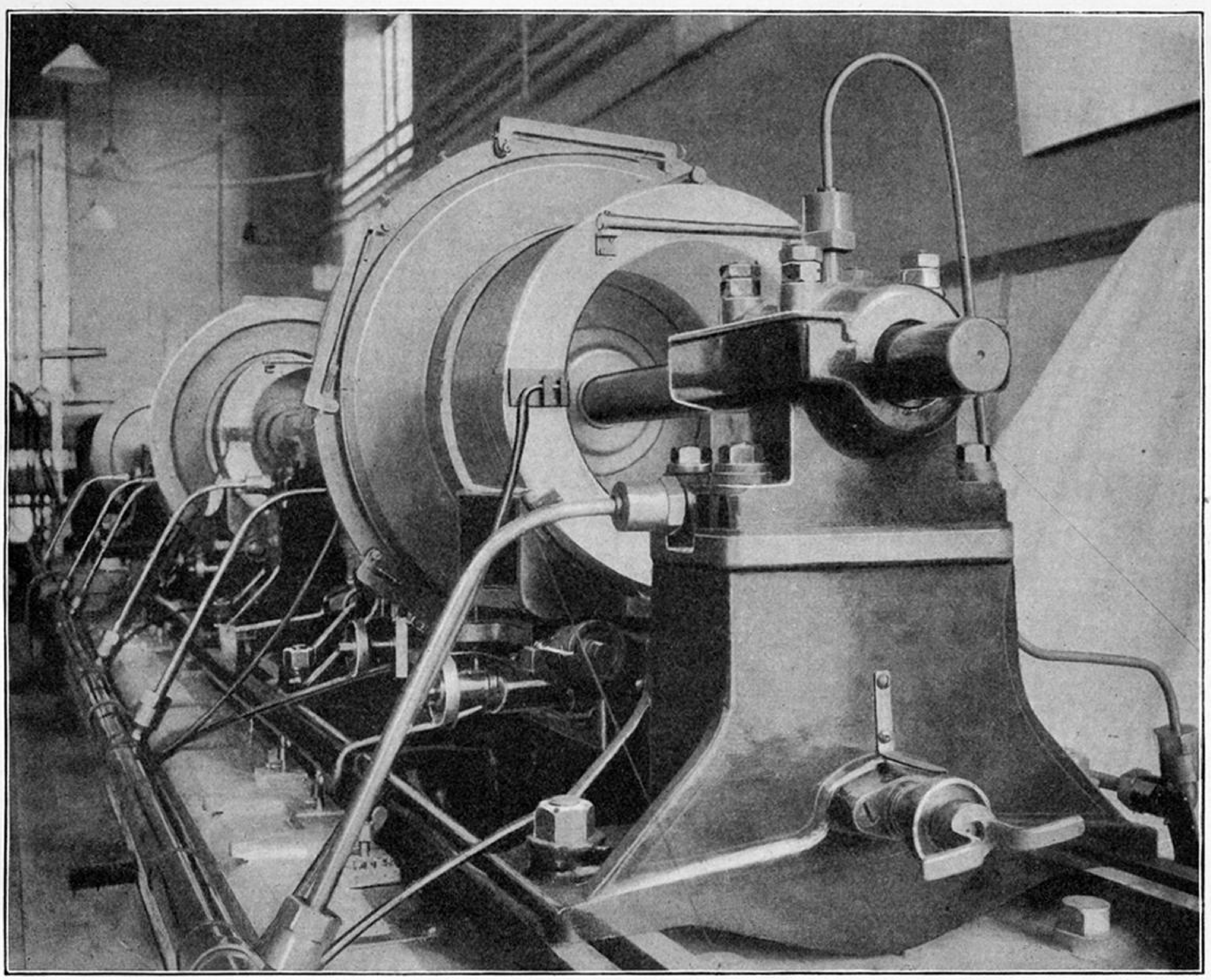

Fig. 3.

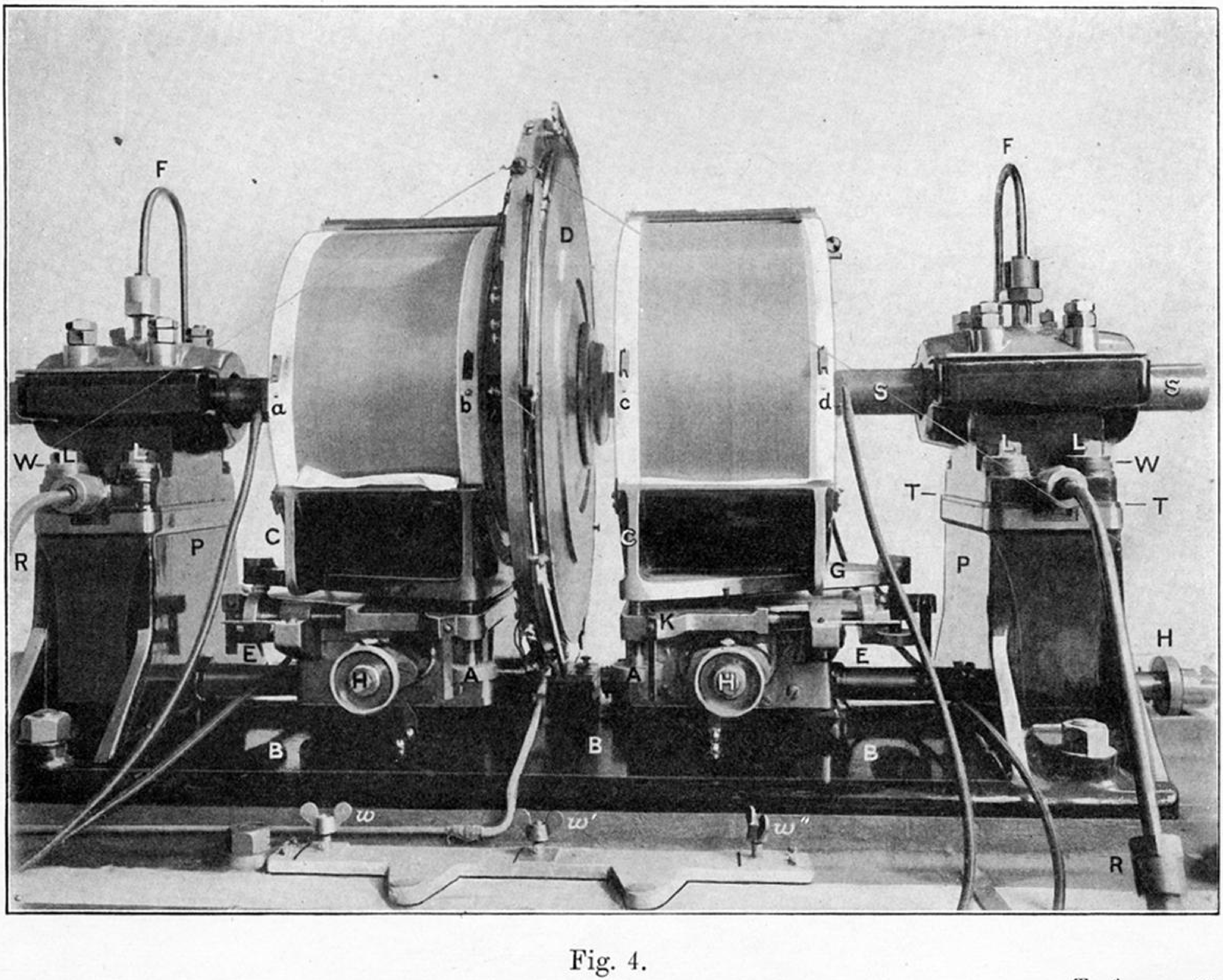

Fig. 4.

To face p. 40.